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The Monetary Policy Implications of Behavioral Asset Bubbles*

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Abstract

I introduce behavioral asset pricing rules into a wider dynamic stochastic general equilibrium framework. Asset price bubbles emerge endogenously within the model. I find that in this model the only monetary policy that would be likely to enhance welfare is a counter-intuitive 'running with the wind' policy. I conclude that the optimal policy is highly dependent on the nature of the behavioral rules that are stipulated. Given that monetary authorities have limited information about the ways in which agents actually behave, a systematic monetary policy response to asset price misalignments is unlikely to enhance welfare.

1 Introduction

The use of asset prices in monetary policy formulation has a long and chequered history. For the majority of history, monetary policy has been inextricably linked to asset prices. The value of money was tied either to the value of precious metals or to the value of other currencies almost continuously up until the final decades of the twentieth century.

However, a near-consensus evolved within the literature over the latter decades of the twentieth century. It held that monetary policy should respond to expected inflation and possibly to the

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output gap, but should not be directly influenced by asset price movements. The intuition for this view is straightforward. In a world characterized by rational expectations, asset prices are simply equal to the discounted sum of the expected future returns to holding those assets. Asset prices simply reflect other fundamental variables in the economy and so monetary policy cannot be improved by targeting asset prices over and above the extent to which it targets those fundamental variables, namely expected inflation and output.

There has been some dissent to this view (for example Cecchetti et al 2000) and, furthermore, some evidence exists to suggest that central banks do indeed take asset price movements into consideration when setting interest rates (Mishkin 2007, Cecchetti et al 2000). Unsurprisingly, most of this dissent is premised on the idea that asset prices may not be a perfect reflection of the economy's fundamentals, that irrational bubbles may exist.

The existing literature on the implications of asset price bubbles for monetary policy, which I briefly review below, fails to reach a consensus on whether or not monetary policy should target such bubbles. In this paper, I intend to contribute to this debate by considering the effects of asset price targeting within a dynamic stochastic general equilibrium (DSGE) model in which expectations of future asset prices are based on simple heuristical rules, as suggested by much of the behavioral finance literature.

This represents a significant innovation when compared to previous contributions to the debate, which have been based either on highly stylized models or on more highly-specified models, but which treat the asset price bubble as an exogenous process.

I find that with the asset bubble endogenized within the model, monetary policy can have perverse effects on the development of the bubble. This observation suggests that the monetary authorities should be weary of targeting asset prices. Indeed, I conclude that any systematic targeting of asset prices is likely to diminish welfare.

In the next section, I will briefly review the existing literature. Subsequently, I will go on to develop a DSGE model in which asset prices are biased by forecasting which is based on the type of heuristical rules suggested by Frankel and Froot (1990) and developed by numerous other authors. I will proceed to analyze the results from the model, and to draw some policy conclusions.

2 Related Literature

In his now infamous speech to the American Enterprise Institute, the then Federal Reserve chairman Alan Greenspan posed a question very similar to the one I am now attempting to answer:

> But how do we know when irrational exuberance has unduly escalated asset values, which then become subject to unexpected and prolonged contractions as they have in Japan over the past decade? And how do we factor that assessment into monetary policy? (Greenspan 1995)

These comments were, at the time, a rare reflection on the potential importance of asset mis-pricing to monetary policy. The slumps in world stock markets which have followed, in the early 2000s and during the recent credit crunch, have strengthened the view that asset price bubbles exist as an empirical fact. However, the literature relating to the subject is still relatively sparse.

Bernanke and Gertler (2000) provide the original investigation into the implications of asset price bubbles for monetary policy. They incorporate an exogenous asset bubble into their financial accelerator model (Bernanke, Gertler and Gilchrist 1999). The bubble is exogenously determined by a simple probabilistic rule. The bubble grows until such time as it bursts, but the expected discounted value of the bubble decays over time. When the bubble bursts, the asset price goes instantaneously back to the fundamental value.

Bernanke and Gertler simulate their model under two alternative monetary policy rules. The first of these rules has the interest rate responding only to inflation, but under the second rule policy also reacts to the lagged asset price. They conclude that the best policy is to focus aggressively on inflation and ignore asset prices, in that this policy achieves the lowest variance of output and inflation. Their simulations show that a monetary policy rule which accommodates inflation but responds to asset prices actually leads to a decline in output and inflation during a positive bubble. The rise in interest rates in response to the bubble drives down fundamental values to a greater extent than the bubble stimulates them. When the monetary policy rule aggressively targets inflation, they find that adding in the response to asset prices makes little difference, though what difference it does make is still destabilizing.

Cecchetti et al (2000) reach very different conclusions from the same model. They criticize Bernanke and Gertler for considering too narrow a set of policy rules and for failing to consider different parameterizations of the New Keynesian Phillips' curve. They report the results of simulations of the model in which they loosen these restrictions. In particular, they consider:

1. Taylor rules which include the output gap, and which allow for interest rate smoothing.
2. policy rules that react to asset mis-pricing rather than to the asset price itself. In other words, they assume that the central bank can distinguish between asset price movements caused by changes to the fundamentals, and those which are caused by a bubble.
3. the implications of making agents more or less backward looking in their wage setting. In other words, they vary the weights on past inflation and future expected inflation in the New Keynesian Phillips' curve.

They report that in the majority of cases, it is optimal for interest rates to respond to asset mis-pricing.

They further criticize Bernanke and Gertler on the basis that both the bubble's size and duration, and the level of leverage in the economy are treated as exogenous. They argue that when private agents expect the monetary authorities to 'prick' a bubble, the bubble is less likely to appear in the first place. Alternatively, if the bank can tighten policy in the formative stage of the bubble it will mitigate the worst excesses that might otherwise occur. The authors also contend that if it is known that the monetary authorities will react to asset prices, then firms and households will react to stock market buoyancy by reducing their leverage, and this will dampen the effect of the financial accelerator.

Bordo and Jeanne (2002) suggest that the best way to think of asset price targeting is as costly insurance against financial crisis. In their highly stylized model, they incorporate a financial shock whose distribution depends on firms' indebtedness. The justification for the endogeneity of this shock is similar to Bernanke and Gertler's explanation of the financial accelerator. It lies in the fact that financial intermediaries rely on collateral to reduce financial frictions. Collateral in turn is driven by asset prices. Given that monetary policy can affect asset prices, and thereby debt accumulation, it also affects the probability of a damaging financial shock. A proactive monetary policy can thus prevent a credit crunch from emerging in the future. However, such a policy incurs a cost in terms of sacrificing short-run macroeconomic objectives.

They find that the optimal monetary policy depends decisively upon the optimism of the private sector. When optimism is low, firms do not leverage themselves very highly, so the risk of a credit crunch is low, and the cost of a proactive policy is not worth bearing. As the private sector becomes more optimistic they increase their leverage and the probability of a credit crunch increases. It becomes worthwhile to insure against that risk with a proactive monetary policy. However, as optimism increases, there is also an increase in the cost of the proactive monetary policy. The cost increases because the more optimistic private agents are, the greater the interest rate that needs to be set to curb their indebtedness. At some point, the cost associated with distorting monetary policy becomes so high that it no longer pays to insure against the credit crunch.

In this way, Bordo and Jeanne conclude that there is no simple rule as to how central banks should respond to asset prices. The optimal policy depends on the economic circumstances in a complex, non-linear way that cannot be represented in a straightforward Taylor rule.

Bean (2004) examines the effects of targeting asset prices within a simple New Keynesian model. His key conclusion is that expectations of future policy actions are at least as significant as current policy in preventing asset booms and busts. In his model, credit crunches occur with a given probability, but their severity depends upon the level of indebtedness in the economy. In the model, higher interest rates reduce capital formation and associated indebtedness, but the higher interest payments exactly offset this so that the output cost of a credit crunch is unaffected. In this way, current monetary policy does not have any impact on the severity of a credit crunch. However, monetary policy can effect the severity of future credit crunches through its impact on future expected output, and therefore on current capital accumulation and leverage. Hence, a central bank may find it optimal to use monetary policy commitments to limit the build up of leverage in the economy. The optimal commitment is in fact to stabilize output by less than the discretionary optimum when a credit crunch occurs. This counter-intuitive result arises because, by committing to a larger output cost if a credit crunch does occur, the central bank is disciplining private agents to limit their indebtedness.

In summary, a number of themes recur within the literature:

1. There is great difficulty in identifying whether asset price movements are driven by changes in the fundamentals or by noise trading. It is only with the benefit of hindsight that bubbles

become recognizable. For many authors (see for example Greenspan 2002) this provides an overwhelming reason for not attempting to target asset prices. Cecchetti et al (2000), on the other hand, make an analogy between the concept of a fundamental asset price and the concept of potential output. They argue that measuring asset mis-pricing is of a similar complexity as measuring the output gap.

2. The macroeconomic consequences of bubbles are relatively mild in the absence of some kind of financial accelerator effect. As Bean (2004) states, "if the only macroeconomic consequences of booms and busts in asset prices were via conventional wealth effects on aggregate demand, then they would constitute little more than a nuisance to monetary policy makers". It is only when falling asset prices combine with financial market frictions to cause credit rationing and credit crunches that significant welfare losses occur.

3. A number of authors argue that the magnitude of the monetary policy response that would be needed to correct for a bubble would risk causing serious harm to the real economy. Greenspan (2002) provides a selection of empirical evidence that suggests that the response of asset prices to monetary policy is weak. Assenmacher-Wesche and Gerlach (2008) estimate VARs in order to assess the responses of equity and house prices to monetary policy across 17 different countries. They concur that using monetary policy to offset asset price movements in an attempt to guard against financial instability may have large effects on economic activity.

4. Conversely, Bean (2004) highlights the way in which a commitment to future policy may have significant effects on the expectations, and hence the behavior, of the private sector. Such commitments, if they are effective in preventing bubbles from occurring, may never actually have to be acted upon. Cecchetti et al (2000) illustrate this in a simulation of the Bernanke and Gertler model. They compare Taylor rules with and without a response to asset prices. Although the asset targeting rule involves a larger response ex-ante to bubbles, ex-post the monetary policy response is smaller because private agents fully expect the central bank's response, and so bubbles do not grow as large.

5. Even if it is appropriate to target asset mis-pricing, the timing of monetary policy poses significant difficulties. The lags in the transmission of monetary policy mean that it may be counter-productive to respond to a bubble with a monetary tightening. If the bubble bursts

of its own accord, just as the monetary tightening takes effect, then the economy will be hit simultaneously by two deflationary forces. Gruen et al (2003) show that the informational requirements for implementing an asset price targeting policy are particularly stringent when these timing considerations are taken into account.

In the remainder of this paper I will present a model which will attempt to address the final three themes highlighted above. Much has been said on the first theme, and though I will return to the issue of measuring mis-pricings in the conclusion, this is not an issue for theoretical modelling. As far as the second theme is concerned, the importance of financial accelerator affects is well established and relatively uncontroversial. Again, therefore, I will not concern myself with this issue in the model that follows, but will return to the issue in my conclusion.

My main concern in what follows is to provide a new perspective on the asset price targeting debate. All of the models discussed above either treat the bubble process as exogenous or use some simple but poorly specified construct to endogenize the bubble process. My main contribution is to use a specific behavioral framework to generate an endogenous bubble.

A further contribution is that I use a fully specified dynamic stochastic general equilibrium (DSGE) model to assess different policy rules. This allows me to do a full welfare analysis, rather than having to resort to ad hoc assessments using central bank loss functions.

3 The Model

The model I use includes all of the usual features of a standard New Keynesian DSGE model (see, for example, Canzoneri, Cumby and Diba 2007). Le, Minford and Wickens (2008) have shown that a model of this general type, with suitable real rigidities, can fit the business cycle behaviour of output, inflation and nominal interest rates for the US economy over the period since the mid-1980s.

The innovation in the model is that I do not impose rational expectations throughout. I maintain the assumption of rational expectations in the goods and labour markets, but decisions in the asset market are governed by simple heuristical forecasting rules. This is consistent with the main contention of the behavioral finance literature. The suggestion is that asset markets are prone to uncertainty and speculation in a way in which goods markets and labour markets are not. For this reason a more complex specification of the forecasting rules employed in asset markets is needed, rather than a simple appeal to rationality.

In this model agents base their expectations of future asset prices on a choice between two simple heuristical rules. They choose between a chartist rule and a fundamentalist rule, depending on the past profitability of the rules. This approach to modelling asset prices was first suggested by Frankel and Froot (1986), and has been shown to be effective in explaining the time series characteristics of asset prices. De Grauwe and Grimaldi (2006) show that a portfolio choice model in which agents switch between rules in this way gives rise to the fat tails, excess kurtosis and GARCH properties which are evident in real world asset price series. Ap Gwilym (2009) finds that such a model cannot be rejected as the data generating process for the FTSE All-Share index.

Here, I will provide a brief overview of the model, concentrating attention on the behavioral forecasting mechanism. For a fuller exposition of the model, see appendix A.

The model economy consists of a set of households, a set of firms and a bond-issuing government.

The households derive an income by providing a differentiated labour service to the firms, for which they set a wage rate via a Calvo mechanism. Labour has a utility cost whilst the consumption of a mixed bundle of output increases utility. Each household maximizes its intertemporal utility, transferring wealth across time by holding government bonds or through capital ownership.

Each firm sets the price for its differentiated output via a Calvo contract. They maximize profits subject to a productivity constraint, and pay a wage to labour and a rent to their capital owners.

I use the artifice of a perfectly competitive bundler to transform the differentiated output of the firms into a homogenous output-bundle which is consumed by the households or reinvested as capital. I use a similar bundler to transform the differentiated labour into a homogenous labour-bundle which is used in the productive process by the firms.

The government's only role in this model is as a bond issuer. I assume that the government sets the nominal interest rate according to a Taylor rule which includes a response to the most recent asset mis-pricing. The government supplies as many bonds as are demanded at this interest rate.

I assume that agents operating in the capital markets choose between two simple heuristical rules when forecasting future asset prices. At the beginning of each period chartist and fundamentalist forecasts of the asset price this period and next are formed. The forecasts of this period's asset price then determine the actual asset price via a bargaining process. The forecasts of next period's asset price imply a particular expected return on capital.

The chartist forecasts of the present and next period asset price are a simple extrapolation of the historical price series:

$$E_{c,t}\left(q_t\right) = q_{t-1} + \chi_c(q_{t-1} - q_{t-2}) + \chi_c^2(q_{t-2} - q_{t-3}) + ... \tag{1}$$

$$E_{c,t}\left(q_{t+1}\right) = E_{c,t}\left(q_t\right) + \chi_c(E_{c,t}\left(q_t\right) - q_{t-1}) + \chi_c^2(q_{t-1} - q_{t-2}) + ... \tag{2}$$

It is arguable as to whether such heuristical rules should be specified in real or nominal terms. I choose real terms on the basis that, in this paper, I am attempting to address the issue of asset market bubbles, and hence I want to avoid the issue of money illusion.

The fundamentalist forecast is that the asset price will move back towards its fundamental value, q_t^*, during the next period, unless it is already close to the fundamental, defined by the bounds $\pm C$:

$$\begin{aligned} E_{f,t}\left(q_t\right) &= q_{t-1} - \chi_f(q_{t-1} - q_t^*) && \quad \textit{where } |q_{t-1} - q_t^*| > C \\ &= q_{t-1} && \quad \textit{where } |q_{t-1} - q_t^*| \leqslant C \end{aligned} \tag{3}$$

$$\begin{aligned} E_{f,t}\left(q_{t+1}\right) &= E_{f,t}\left(q_t\right) - \chi_f(E_{f,t}\left(q_t\right) - q_t^*) && \quad \textit{where } |q_{t-1} - q_t^*| > C \\ &= E_{f,t}\left(q_t\right) && \quad \textit{where } |q_{t-1} - q_t^*| \leqslant C \end{aligned} \tag{4}$$

We can think of C as an uncertainty bound. Fundamentalists recognize the uncertainty inherent in their modelling of the fundamental price and so they only take an active trading position when the actual price is significantly different from the fundamental value. If the actual price is already close to fundamental (i.e. within the band defined by C) then they predict no change in the actual price.

The contention of behavioral economics is that the level of complexity in the real world makes it impossible for agents to fully comprehend the markets in which they trade. In such a world, the ex-ante use of simple rules such as those in this model may constitute a best response. However, even in a complex world, the ex-post assessment of trading rules is relatively cheap. I therefore impose some limited rationality in the form of an evolutionary switching procedure based on the ex-post profitability of the competing rules. Agents are assumed to assess the ex-post risk adjusted profitability, $\Omega_{i,t}$, of each of the forecasting rules and then select the rule that they will use in the next period. Hence, the proportions of agents using each of the rules develops according to the following identities:

$$wc_t = \frac{\exp(\upsilon\Omega_{c,t})}{\exp(\upsilon\Omega_{f,t}) + \exp(\upsilon\Omega_{c,t})} \tag{5}$$

$$wf_t = \frac{\exp(\upsilon\Omega_{f,t})}{\exp(\upsilon\Omega_{f,t}) + \exp(\upsilon\Omega_{c,t})} \tag{6}$$

The parameter υ represents the propensity with which agents switch between the forecasting rules. $\Omega_{c,t}$ and $\Omega_{f,t}$ are the excess returns over holding bonds associated with following the chartist forecast and the fundamentalist forecast respectively. They are calculated as follows:

$$\Omega_{c,t} = \left[q_{t-1} - q_{t-2}\left(1 + r_{t-1}^{bB}\right)\right].sign\left[E_{c,t-2}\left(q_{t-2}\right) - q_{t-2}\right] \tag{7}$$

$$\Omega_{f,t} = \left[q_{t-1} - q_{t-2}\left(1 + r_{t-1}^{bB}\right)\right].sign\left[E_{f,t-2}\left(q_{t-2}\right) - q_{t-2}\right] \tag{8}$$

where $q_{t-2}\left(1 + r_{t-1}^{bB}\right)$ represents the return to investing funds in bonds and q_{t-1} is the return to investing the same funds in equities. $sign\left[E_{c,t-2}\left(q_{t-2}\right) - q_{t-2}\right]$ takes the value -1 when $E_{c,t-2}\left(q_{t-2}\right) < q_{t-2}$, in which circumstances an agent following the chartist rule would choose to invest in bonds rather than equities, and takes the value +1 when chartists choose equities over bonds. $sign\left[E_{f,t-2}\left(q_{t-2}\right) - q_{t-2}\right]$ is analogous.

The actual asset price is determined via a bargaining process between those who favour the chartist rule and those who favour the fundamentalist forecast:

$$q_t = w_{c,t}E_{c,t}\left(q_t\right) + w_{f,t}E_{f,t}\left(q_t\right) \tag{9}$$

The average expected return on capital that is implied by these behavioral forecasting rules is that which entails that the present asset price is equal to the discounted average future expected asset price. In other words, it is the return, $E_{b,t}r^k_{t+1}$, which satisfies:

$$q_t = \beta \frac{E_t \lambda_{t+1}}{\lambda_t} \left\{ E_{b,t} r^k_{t+1} + \left[w_{c,t} E_{c,t} \left(q_{t+1} \right) + w_{f,t} E_{f,t} \left(q_{t+1} \right) \right] \right\} \tag{10}$$

In this way, we can think of behavioral rules as driving an asset mis-pricing by inducing a bias in the expected future return to capital.

In order to ensure the tractability of the analysis, I limit the sources of randomness in the model to a single productivity shock.

Welfare is assessed directly, using the utilitarian notion that welfare is simply equal to the aggregation of individual utilities.

3.1 Parameterizing the Model

My intention in this paper is to pose questions about the effectiveness of monetary policy as a tool for alleviating the damaging effects of asset price bubbles. It is not to produce a model for calibrating the optimal policy. I, therefore, make no attempt to estimate the model. Instead, I borrow my parameterization of the model from previous work. The parameters for the New Keynesian aspects of the model are taken from Canzoneri et al (2007), whilst those for the behavioral aspects are taken from ap Gwilym (2009). The baseline parameterization that I use is given in appendix C.

4 Solving the Model

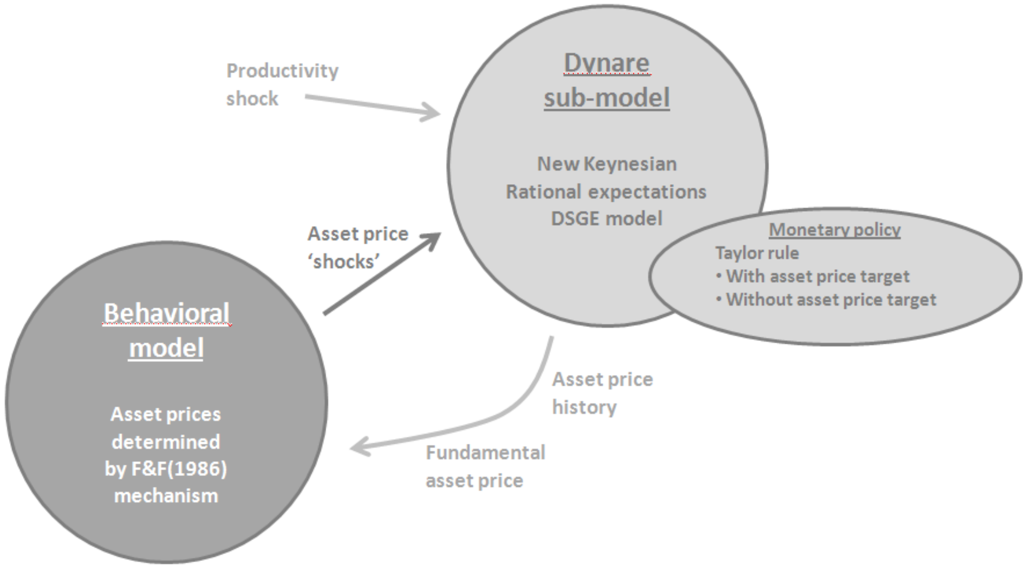
Solving this model presents very significant challenges. I am assuming rationality in the goods and labour markets, which means that agents are forward looking and understand the model, so that all of the equations that describe these markets must hold in both the present period and in their expected terms for all future periods. On the other hand, I am assuming that rationality breaks down in the asset market. In keeping with the spirit of the behavioral finance literature, agents make use of simple heuristical rules to determine what they consider to be a fair price for the asset and also in determining how the asset price will behave in the future. However, due to the complexity of the asset market, agents do not understand the behavior of others in the market, and so the market clearing condition only holds in the present period. It does not hold in its expected future forms.

I can always impose some kind of asset mis-pricing. However, as long as agents understand the mechanism which causes that mis-pricing, and they rationally expect it to persist, then all that happens is that there is a persistent dislocation between the fundamental value of the asset and its price. If future dislocations are fully anticipated then the net effect is that the expected return to asset-holding is unchanged. If this is the case, then there will be no distortions in the wider economy. The key here is that it is biases to the expected return on asset holding, and not the mis-pricing of the asset in and of itself, which drives distortions in the allocation of resources.

My approach to solving the model is to take advantage of well established techniques for solving rational expectations models, and then introduce the non-rational aspects of the model via 'shock' processes. Shocks to the current and future expected asset price drive these variables away from their fundamental values. Of course, these are not shocks in the conventional sense. The behavioral model describes exactly how these deviations from fundamental are determined. As already stated, if the agents in the model understood this behavioral process then they could form a rational expectation of future deviations, and the asset mis-pricing would not effect the wider economy. By introducing these biases to the current and future asset price as unanticipated shocks, they cause a bias to the expected return on capital, and thereby alter the allocations throughout the economy.

Figure 1 summarizes the solution method. I solve the majority of the model as if it were a rational expectations model, using the Dynare pre-processor. The solution to this Dynare sub-model consists of a set of policy and transition functions which describe how each variable is determined by pre-determined variables (initial conditions) and a set of 'shocks'. There are four 'shocks' to the Dynare sub-model. The first is a conventional productivity shock. There are a pair of asset price 'shocks'. One affects the present asset price, driving a wedge between the actual asset price and its fundamental value. The other asset price 'shock' drives a wedge between the expected future asset price and its fundamental value. Combined, these two shocks have the effect of biasing the expected return on capital, which in turn drives distortions in the rest of the model. The fourth 'shock' is the monetary policy response to the asset mis-pricing.

Figure 1: Solving the model



The productivity shock is treated as a random variable. The other three shocks, however, are endogenously determined. The asset price 'shocks' are determined by the behavioral asset pricing rules explained above (equations 1 to 8). The monetary policy response is determined by the Taylor rule discussed below.

Appendix B presents a preliminary analysis of the Dynare sub-model.

5 Results

In order to assess the effect of including an asset price target in the monetary policy rules, I simulate the entire model under different parameterizations of the Taylor rule. The parameterization that I use in the benchmark model is as follows:

$$R^b_{t+1} = 0.01 + \Pi_t + 2.02\left(\Pi_t - \Pi^*\right) + 0.184 \ln\left(\frac{y_t}{y^*}\right) + \zeta_q \ln\left(\frac{q_{t-1}}{q^*_{t-1}}\right)$$

This is based on Canzoneri et al's (2007) estimation of a Taylor rule over the Volcker and Greenspan years as Federal Reserve chairmen (1979 - 2003). Of course, the estimated Taylor rule does not include a response to asset mis-pricing, so $\zeta_q = 0$.

I run 1,000 stochastic simulations of the model under each of several different values of ζ_q. I consider:

1. a 'passive' monetary regime, where there is no response to asset mis-pricings. In other words, $\zeta_q = 0$.
2. a variety of 'active' regimes with different weights, both positive and negative, on the asset mis-pricing. The weights I consider are $\zeta_q = 0.05, -0.05, 0.1, -0.1, 0.5, -0.5, 1$ and -1.

For each individual simulation, I run the model for forty periods. The only exogenous shock is the productivity shock, and this is drawn at random from a normal distribution with mean zero and standard deviation 0.0086[1] for each of the forty periods. I calculate the actual welfare each period and, for the final period, I also calculate the expected future welfare. I then discount these values to get a measure of inter-temporal welfare for each simulation. Averaging this welfare measure across the 1,000 simulations gives a measure of expected welfare under each alternative monetary regime.

As is the case for any measure of welfare, the cardinal units are more or less meaningless. I follow the convention, initiated by Lucas (2003), of calculating and reporting consumption equivalents. The welfare measures for all of the results reported in this paper refer to the proportion of consumption that households would be prepared to give up permanently, holding work effort constant, in order to live in a rational world with no asset bubbles and no associated monetary response.

Table 1 presents the estimation of welfare in the benchmark behavioral model, under different specifications of the Taylor rule. The second column of the table states the loss in welfare relative to the rational model under each monetary regime.

The behavioral model with a passive monetary regime ($\zeta_q = 0$) leads to expected welfare which is equivalent to a loss of 0.177 of one percent of permanent consumption relative to the rational model. We can think of this as the cost of irrationality in the asset markets. I use this value as a basis against which to compare the performance of the alternative monetary policy options.

[1] This parameter comes from an estimate of the 1960 - 2002 US data (with a log linear trend) by Canzoneri et al (2007)

Table 1: Utility cost of various Taylor rules in the benchmark model

Monetary regime $\zeta_q =$	**Utility cost versus rational model** (%age permanent consumption)	**Performance relative to** $\zeta_q = 0$ (%age bias corrected)
0.00	-0.177 %	0.0 %
0.05	-0.187 %	-6.1 %
-0.05	-0.167 %	5.5 %
0.10	-0.199 %	-12.8 %
-0.10	-0.158 %	10.5 %
0.50	-0.357 %	-101.9 %
-0.50	-0.113 %	35.9 %
1.00	-1.083 %	-512.8 %
-1.00	-0.113 %	36.3 %

For example, the behavioral model with a modest 'leaning against the wind' monetary regime of $\zeta_q = 0.05$[2] experiences a loss in welfare relative to the rational world which is equivalent to 0.187 of one percent of permanent consumption. Compared to the passive regime, this represents an exacerbation of the misallocation of resources which irrationality has caused. The welfare loss is 6.1% higher than under the passive monetary regime. This measure of the performance of active monetary policy rules relative to the passive regime is reported in the third column of table 1.

In other words, a negative number in column 3 signals a monetary policy rule that increases the cost of behavioral biases - we can think of this as a monetary policy rule that exacerbates those biases. On the other hand, a positive number in column 3 signals a monetary policy rule that reduces the cost of behavioral biases - we can think of this as a monetary policy rule that corrects for those biases.

The surprising result is that 'leaning against the wind' policies (policies which lead to a monetary tightening when asset prices are above fundamental, and a loosening when they are below fundamental, i.e. $\zeta_q > 0$) exacerbate the cost of behavioral biases. Even more surprisingly, the contrary 'running with the wind' policy, where an asset bubble is met with a monetary loosening, actually ameliorates the effects of the bubble. An explanation of these counter-intuitive results is required, and I will provide this in the next section.

[2] This is the monetary policy response recommended by Cecchetti et al (2000)

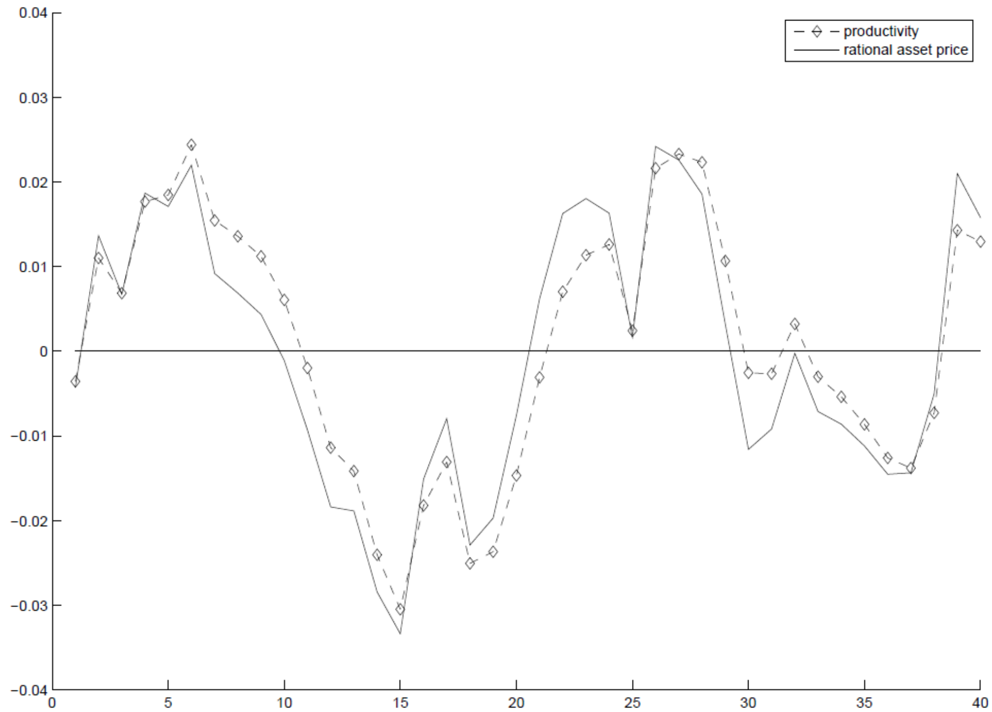
Before providing that explanation, I shall briefly consider the significance of the welfare losses reported. The figures presented in table 1 seem relatively small, but if we compare them to Lucas (2003) they are certainly not trivial. Using US data, Lucas calculates that the welfare cost of fluctuations in consumption around its trend is only about 0.05 of one percent of consumption. Although this model, and the utility function I use, differs from Lucas', this does give us some idea of the significance of the effect of the behavioral biases in the model. Furthermore, there are two reasons to believe that this cost is understated.

1. Firstly, the idea of using future expectations from this model as the basis for welfare comparisons is clearly problematical. The fact that the model is not based on fully rational expectations means that ex-ante (or anticipated) welfare can be inconsistent with ex-post (or realized) welfare. Given that I use expectations to estimate the effect on welfare from period 41 onwards, there are grounds to believe that this measure of welfare underestimates the effect of the behavioral biases on actual welfare. Despite the fact that later periods are more heavily discounted, periods 41 to ∞ still account for around two thirds of inter-temporal welfare.

2. Secondly, as noted earlier, the main costs associated with asset price bubbles in the real world are as the result of credit rationing which often accompanies the bursting of a bubble. In this model, I have no financial market frictions which could cause such a credit crunch. The only way in which asset mis-pricings effect allocations in the real economy is via wealth effects. Hence, the fact that I derive a significant cost to behavioral biases even in the absence of financial accelerator effects is noteworthy.

If my intention in this paper were to try and provide a meaningful estimate of the cost of behavioral biases, then I would clearly need to address both of these issues. In fact, my intention is not to do that, but rather to assess the effectiveness of monetary policy in correcting for behavioral biases. In order to do this, I do not need an accurate estimate of the cost of behavioral biases, I only need for my estimate of that cost to be consistent across monetary policy regimes. For that reason, and given the computing power necessary to accurately measure ex-post welfare, and given the difficulty of introducing financial accelerator effects into a DSGE model, I will postpone the attempt to accurately estimate the costs of behavioral biases until further work.

Figure 2: The productivity process and rational asset price in a simulation of the model



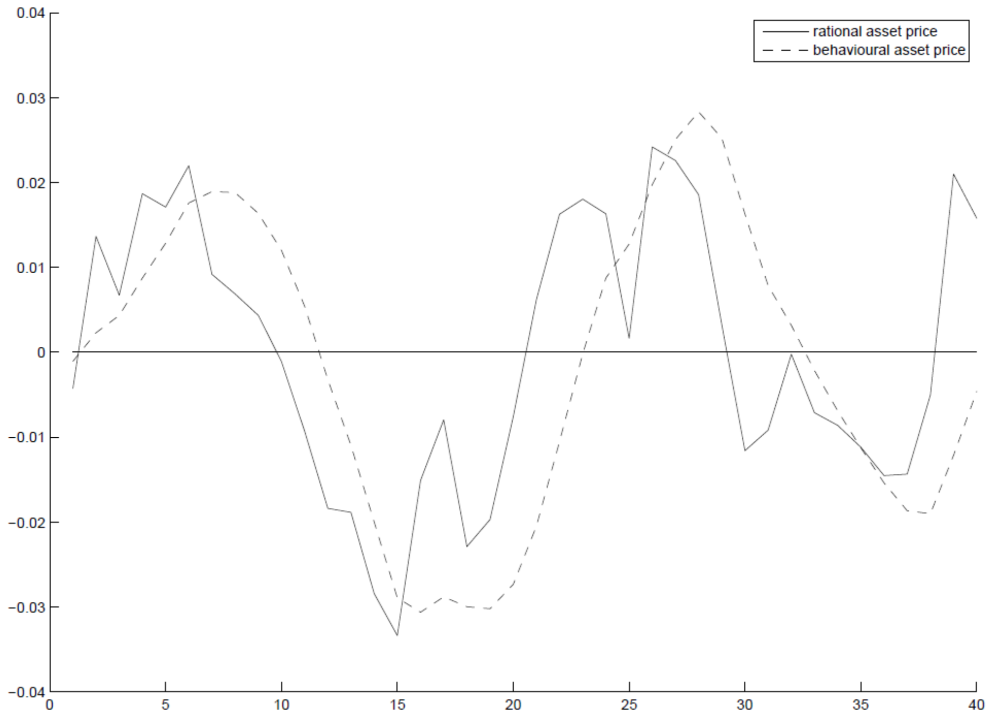
5.1 Intuition for the Result

In order to provide intuition for these results, I consider here a single simulation of the model.

Figure 2 shows the productivity process in this particular simulation. It also shows that the rational asset price mimics, almost exactly, the productivity process[3]. Figure 3 contrasts the asset price in the rational model with that in the behavioral model under a passive monetary regime, based on the same underlying productivity process. We can see that the effect of the behavioral rules is that the asset price becomes slow to react to changes in productivity. This is because initially only fundamentalists react to the change in productivity. Chartists gradually jump on the bandwagon, but are slow to react when there is a turning point in productivity. This type of dynamics coincides with the type of story that is often told to explain asset price bubbles. For example, during the late 1980s and early 1990s there was a revolution in information technology,

[3]The rational asset price illustrated in figure 2 is the asset price that I get when I simulate the model without any behavioural biases. This differs from the fundamental asset price under any simulation of the full model because the behavioural rules affect capital accumulation, and this in turn drives changes in what would then be a rational asset price.

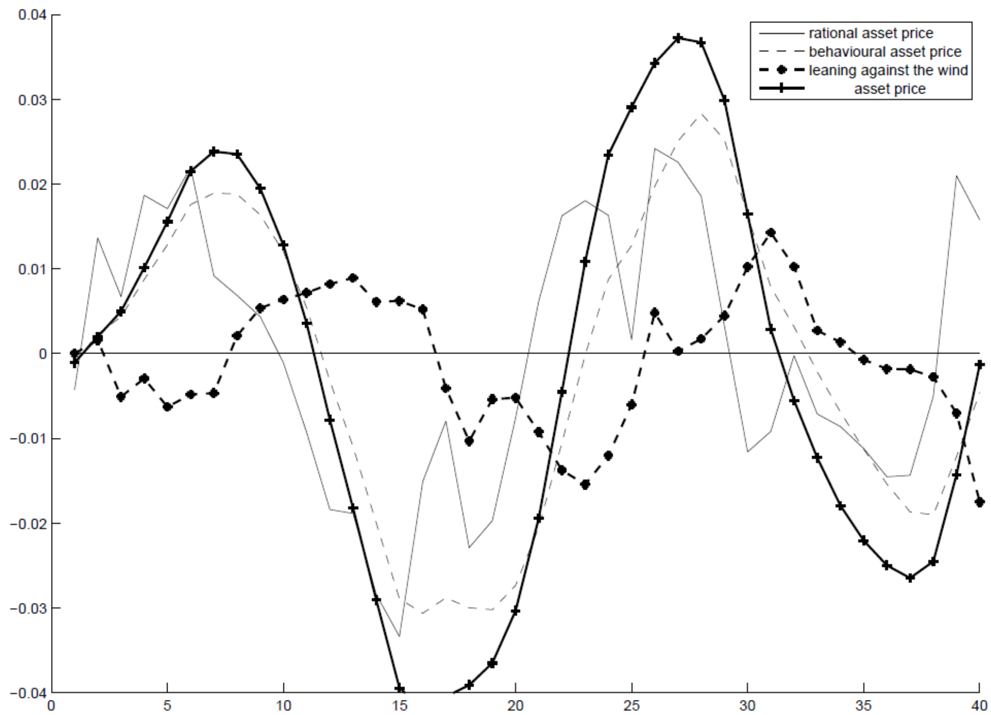
Figure 3: The asset price in the rational model and in the behavioural model with a passive monetary policy



largely based on the growth of the internet, which did actually drive improvements in productivity. A plausible explanation of equity prices through the 1990s might argue that at first there was a slow response to the fundamental changes in productivity that were being driven by technological progress. However, once the response started to take place it accelerated at exactly the same time as the growth rate of productivity began to return to more normal levels. Hence, asset prices began to outstrip their fundamental value in what came to be known as the dot-com bubble. Similar stories have been told about the recent boom in house prices, but this time based on financial rather than informational innovations.

The question now is what effect monetary policy has in the model. Figure 4 illustrates the effect of a 'leaning against the wind' monetary policy, with $\zeta_Q = 0.5$. The dashed line marked with blobs is the monetary policy (interest rate) response to the asset mis-pricing. Monetary policy is expansionary (low interest rate) when the asset price was below fundamental in the previous period and it is contractionary (high interest rate) when the asset price was above fundamental in

Figure 4: The asset price in the behavioural model with a 'leaning against the wind' monetary policy



the previous period. With this monetary policy response, the asset price is described by the solid line marked with pluses. As we can see, in periods 1 to 7 of this simulation the monetary policy response has the effect of driving the asset price closer to its fundamental value (approximated by the rational model). However, this means that when productivity growth falls off, in period 8, the chartist rule imparts greater momentum onto the asset price than it did under the passive monetary regime. Therefore, when the productivity process reaches a turning point, the bubble that develops is more pronounced. To return to my previous analogy, if the monetary authorities had recognized the under-pricing of equities during the technological boom of the early 1990s, and had relaxed monetary policy in response to the mis-pricing there would have been two consequences. In the short-run the under-pricing would have been reduced. However, the faster growth in equity prices that that would entail would have resulted in a greater momentum in asset prices when technological progress diminished in the latter half of the decade. Hence, the dot-com bubble would have been more pronounced than what actually occurred.

Figure 5: The asset price in the behavioural model with a 'running with the wind' monetary policy

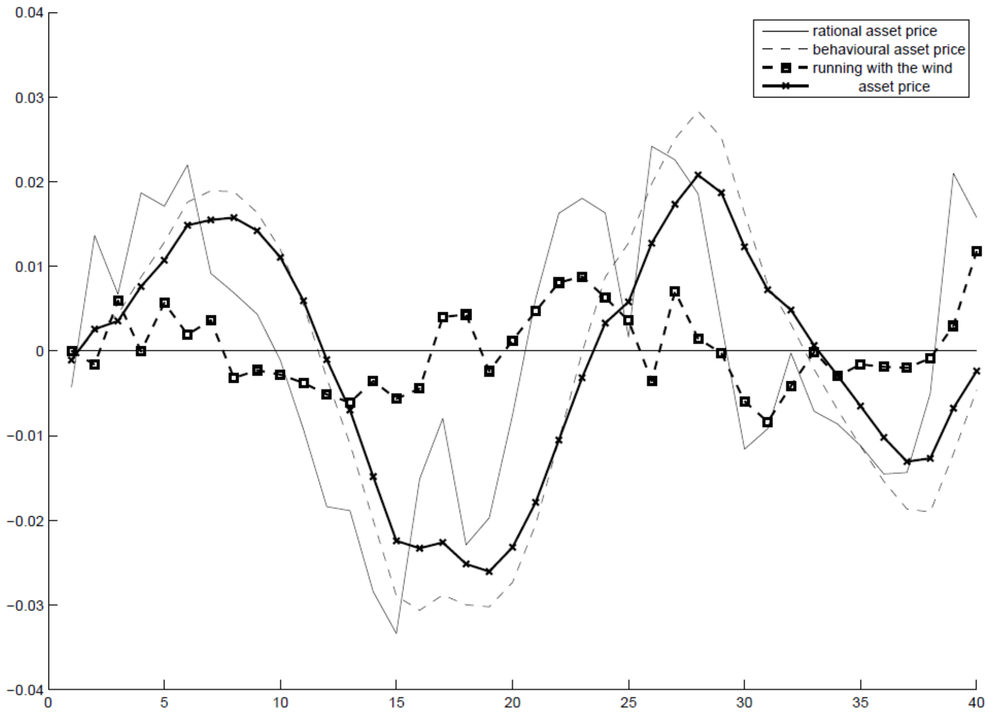


Figure 5 illustrates the effect of a 'running with the wind' monetary policy, with $\zeta_Q = -0.5$. The dashed line marked with squares is the interest rate response to the asset mis-pricing. Monetary policy is contractionary when the asset price was below fundamental in the previous period and it is expansionary when the asset price was above fundamental in the previous period. With this monetary policy response, the asset price is described by the solid green line. The effect here is the reverse of the 'leaning against the wind' policy. During periods 1 to 7 the asset price drifts further away from its fundamental value. However, when productivity growth begins to fall the momentum in the chartist forecast is less than under other policy specifications, and so the resulting bubble is less pronounced.

In this model bubbles appear when turning points in the productivity process occur. The crucial factor in how large those bubbles turn out to be is the rate of change of the asset price in the periods prior to any turning point. There is a momentum inherent in the chartist forecasting rule, and this momentum is greater when the rate of change in the asset price is higher. Since a 'leaning against the wind' policy tends to promote rapid changes in asset prices, it also promotes greater momentum in the chartist rule, and more pronounced bubbles.

5.2 Robustness Testing

In appendix D, I report the results of some robustness testing exercises. Each table in the appendix is equivalent to table 1, but I have altered one of the parameters in the behavioral rules. I consider different values for the propensity to switch between the forecasting rules. I also consider different values for the uncertainty band in the fundamentalist rule, C.

The only case in which a 'leaning against the wind' policy increases welfare is when the switching parameter, υ, is very large. Even in this case only a small response is beneficial, whilst an aggressive policy is extremely harmful.

6 Conclusions

The main result of the model that I have developed in this paper is that 'leaning against the wind' monetary policies are counter-productive whilst 'running with the wind' policies can ameliorate the effects of behavioral biases. This result is clearly model specific. It relies on the particular characteristics of the behavioral forecasting rules on which the model is based. Although it has been shown that these simple heuristical rules can account for historical asset price dynamics (ap Gwilym 2009), I do not want to claim that they are a realistic description of real world financial markets. I, therefore, need to be cautious in drawing policy conclusions from this model.

What is clear is that 'leaning against the wind' monetary policies cannot be relied upon to correct for behavioral biases, and in some cases will cause serious harm. It is not so clear that we should be supporting systematic use of 'running with the wind' policies. It is likely that this result is very model specific. The issue here is that the dynamics of the bubble, and the behavioral underpinnings of those dynamics, are extremely important in determining the most relevant policy response. The big problem, of course, is that the monetary authorities do not have much understanding of that behavior.

The argument in favour of including asset price misalignments in the Taylor rule is summarized by Cecchetti et al (2000) as follows:

> "when significant asset price misalignments occur, they help to create an undesirable instability in inflation and/or employment that may be exacerbated when the misalignment is eventually eliminated. A pre-emptive policy approach therefore will tend to limit the build-up of such asset misalignments and macroeconomic imbalances, and would also limit the required eventual correctionand thereby the medium-term variability of inflation and output."

In this paper, I have shown that pre-emptive policy cannot be relied upon to limit misalignments across time. In the case of the model presented above, the measures required to reduce present misalignments have the effect of increasing future misalignments. Cecchetti et al's intuition that a pre-announced, systematic 'leaning against the wind' policy must drive asset prices closer to their fundamental does not necessarily hold in a world where expectations are not formed rationally.

It is important to note some caveats to what I have said so far. Firstly, my model ignores any financial accelerator effects that may exist. These would likely exacerbate the costs of sub-rational behavior. However, they will also serve to add further complexity to the picture.

Secondly, my model does not allow for any effect caused by expectations of future monetary policy. The model does not allow for the fact that a pre-commitment to a 'leaning against the wind' might cause private agents to hedge against mis-pricings by shunning chartist rules. However, we know that if there is an asset mis-pricing, it must be as the result of some irrationality in the market. If such irrationality exists then it is unclear as to why it would not prevent agents from responding optimally to policy pre-commitments. In Calvo-type models, monetary policy can correct for nominal rigidities by keeping inflation and expectations of inflation equal, through a commitment to an inflation target. In such models, rationality reigns and the monetary authorities are able to take advantage of this in predicting how private expectations will react to future monetary policy. As soon as we loosen the requirement for rationality, it becomes difficult for the monetary authorities to predict how policy affects private sector expectations.

Thirdly, it is important to note that in my analysis, I have only considered systematic monetary policy rules. It may well be possible that a one off monetary tightening or loosening could be effective in increasing welfare if the monetary authorities are able to identify turning points in the productivity process.

Bubbles can only exist in a world where agents are less than fully rational. Furthermore, the dynamics of bubbles are dependant upon the nature of that irrationality. The relevant policy response to a bubble depends upon the particular nature of the behavioral reaction of the market. We have only a very rudimentary understanding of such behaviors. What we do know suggests that such behaviors are complex and cause discontinuities in the relationships between different variables.

For all these reasons, it is likely that any attempt by central banks to try to influence market psychology is likely to have highly unpredictable outcomes. A systematic monetary policy response to asset mis-pricings is unlikely to enhance welfare because monetary authorities lack the full information necessary to implement an effective policy.

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APPENDICES

A The Model

A.1 Household Maximization

There is a continuum of households indexed by h across the unit interval. Each household maximizes its intertemporal utility function,

$$U_{h,t} = E_t \sum_{s=t}^{\infty} \beta^{s-t} \left\{ \ln(c_{h,s}) - \frac{l_{h,s}^{1+\phi}}{1+\phi} \right\}$$

where $c_{h,t}$ is the household's real consumption of the composite good, y_t, and $l_{h,t}$ is its differentiated labour supply. I consider a cashless economy in which households can transfer their wealth from one time period to another by holding government bonds, $b_{h,t}$, or investing in capital, $k_{h,t}$. Government bonds are one period, paying a pre-announced gross nominal return of $(1 + R_t^b)$. Capital is bought at the price P_t and has a rental return R_t^k each subsequent period. Capital depreciates at the rate δ and there is a capital adjustment cost. Hence, each household faces the budget constraint:

$$k_{h,t} R_t^k + b_{h,t}(1 + R_t^b) + l_{h,t} W_{h,t} = c_{h,t} P_t + i_{h,t} P_t + b_{h,t+1}$$

and the law of motion of capital is given by:

$$k_{h,t+1} = (1-\delta) k_{h,t} + i_{h,t} - \frac{\nu}{2} \left(\frac{i_{h,t}}{k_{h,t}} - \delta \right)^2 k_{h,t}$$

where the final term represents the cost of capital adjustment.

Due to nominal rigidities in the labour market, only a proportion of households, $(1-\omega)$, are free to adjust their wage. They choose the wage which maximizes their utility across the states of nature for which that wage rate will hold. The remainder of the households simply update their last period wage by the steady state gross inflation rate, $(1 + \Pi)$.

Households sell their differentiated labour in a monopolistically competitive market to a perfectly competitive bundler. The bundler combines the labour of the various households into aggregate labour which is employed by the firms. The bundling technology is a Dixit-Stiglitz aggregator:

$$l_t = \left[\int_0^1 l_{h,t}^{\frac{\gamma-1}{\gamma}} dh \right]^{\frac{\gamma}{\gamma-1}}$$

The bundler's cost minimization implies that each household faces the following demand for their labour service:

$$l_{h,t} = \left(\frac{W_{h,t}}{W_t} \right)^{-\gamma} l_t$$

Households which are free to optimize in period t choose the wage rate, $W_{h,t}^*$, which maximizes utility across the states of nature for which that wage rate will hold, subject to the labour demand curve, the budget constraint and the law of motion of capital. In other words, it maximizes:

$$E_t \sum_{s=t}^{\infty} (\omega\beta)^{s-t} \left\{ \begin{array}{c} \ln(c_{h,s}) - \frac{\left[\left((1+\Pi)^{s-t} W_{h,t}^* \right)^{-\gamma} W_s^{\gamma} l_s \right]^{1+\phi}}{1+\phi} \\ -\mu_{h,s} \left[k_{h,s+1} - (1-\delta) k_{h,s} - i_{h,s} + \frac{\nu}{2} \left(\frac{i_{h,t}}{k_{h,t}} - \delta \right)^2 k_{h,t} \right] \\ +\Lambda_{h,s} \left[\begin{array}{c} k_{h,s} R_s^k + \left((1+\Pi)^{s-t} W_{h,t}^* \right)^{1-\gamma} W_s^{\gamma} l_s + \\ b_{h,s} \left(1 + R_s^b \right) - c_{h,s} P_s - i_{h,s} P_s - b_{h,s+1} \end{array} \right] \end{array} \right\}$$

Therefore:

$$W_{h,t}^* = \left[\frac{\gamma}{\gamma - 1} \frac{E_t \sum_{s=t}^{\infty} (\omega\beta)^{s-t} (1+\Pi)^{-\gamma(1+\phi)(s-t)} W_s^{\gamma(1+\phi)} l_s^{1+\phi}}{E_t \sum_{s=t}^{\infty} (\omega\beta)^{s-t} \Lambda_{h,s} W_s^{\gamma} l_s (1+\Pi)^{(1-\gamma)(s-t)}} \right]^{\frac{1}{1+\gamma\phi}}$$

When wages are fully flexible (i.e. $\omega = 0$ and $W_t = W_{h,t}^*$) this reduces to $W_{h,t}^* = \frac{\gamma}{\gamma-1} \frac{l_t^{\phi}}{\Lambda_{h,t}}$. In other words, the wage is a mark up over the disutility of work.

The aggregate wage is given by:

$$\begin{aligned} W_t &= \left[\int_0^1 W_{h,t}^{1-\gamma} dh \right]^{\frac{1}{1-\gamma}} \\ &= \left[\sum_{i=0}^{\infty} (1-\omega) \omega^i \left(\Pi^i W_{h,t-i}^* \right)^{1-\gamma} \right]^{\frac{1}{1-\gamma}} \\ &= \left[(1-\omega) W_{h,t}^{*1-\gamma} + \omega \Pi^{1-\gamma} W_{t-1}^{1-\gamma} \right]^{\frac{1}{1-\gamma}} \end{aligned}$$

A.2 Firm Optimization

There is a continuum of retail firms indexed by f across the unit interval. Each firm hires bundles of labour at the aggregate wage rate, W_t. They hire capital from the households in a perfectly competitive factor market at the rental rate of capital, R_t^k. The firms make the decision of how much labour, $l_{f,t}$, and how much capital, $k_{f,t}$, to employ; and thus how much output to produce, $y_{f,t}$. They sell their output in a monopolistically competitive market at the price, $P_{f,t}$, and are constrained by production technology.

Each firm chooses an input mix to maximize profits:

$$y_{f,t}P_{f,t} - k_{f,t}R_t^k - l_{f,t}W_t$$

subject to its production function:

$$y_{f,t} = z_t k_{f,t}^{\alpha} l_{f,t}^{(1-\alpha)}$$

Note that the technology, z_t, is common across all firms. It is assumed to follow a simple stochastic autoregressive process:

$$\ln(z_t) = \varrho \ln(z_{t-1}) + \varepsilon_t \qquad \varepsilon_t \sim N(0, \sigma_\varepsilon)$$

The firm's optimal inputs are:

$$l_{f,t} = \frac{(1-\alpha)y_{f,t}P_{f,t}}{W_t}$$

$$k_{f,t} = \frac{\alpha y_{f,t}P_{f,t}}{R_t^k}$$

When firms charge different prices, the optimal level of inputs varies across firms. However, the optimal capital to labour ratio is constant across firms:

$$\frac{k_{f,t}}{l_{f,t}} = \frac{\alpha}{(1-\alpha)}\frac{W_t}{R_t^k}$$

Because of this symmetry, marginal cost is also constant across firms. It can be derived as:

$$MC_{f,t} = \frac{W_t^{1-\alpha} {R_t^k}^{\alpha}}{z_t \alpha^\alpha (1-\alpha)^{(1-\alpha)}}$$

Calvo-type nominal rigidities in the goods market, entail that in each period only a randomly chosen fraction, $(1-\eta)$, of the firms are free to reset their prices. These firms set new prices taking their respective demand curves as given. The remainder of the retail firms cannot reoptimize, but adjust their price by the steady state inflation, $(1+\Pi)$.

If a firm has the opportunity to reset its price then it chooses the new price, $P^*_{f,t}$. The general price level is:

$$\begin{aligned} P_t &= \left[\int_0^1 P_{f,t}^{1-\theta} df\right]^{\frac{1}{1-\theta}} \\ &= \left[\sum_{i=0}^{\infty}(1-\eta)\eta^i \left(\Pi^i P^*_{f,t-i}\right)^{1-\theta}\right]^{\frac{1}{1-\theta}} \\ &= \left[(1-\eta)P^{*\,1-\theta}_{f,t} + \eta\Pi^{1-\theta}P^{1-\theta}_{t-1}\right]^{\frac{1}{1-\theta}} \end{aligned}$$

I assume the artifice of a perfectly competitive goods bundler employing Dixit-Stiglitz technology. Each individual firm, therefore, faces the demand curve:

$$y_{f,t} = \left(\frac{P_{f,t}}{P_t}\right)^{-\theta} y_t$$

In periods when the firm gets the opportunity to choose a new price, it chooses the price which maximizes its expected discounted future stream of profits across the states of nature for which that price will hold. In other words, it maximizes:

$$E_t \sum_{s=t}^{\infty}(\eta\beta)^{s-t}\frac{\Lambda_s}{\Lambda_t}\left[y_s P_s^{\theta}\left\{(1+\Pi)^{s-t}P^*_{f,t}\right\}^{1-\theta} - TC_{f,s}\right]$$

This yields the optimal price:

$$P^*_{f,t} = \frac{\theta}{\theta-1}\frac{E_t\sum\limits_{s=t}^{\infty}(\eta\beta\left(1+\Pi\right)^{-\theta})^{s-t}\Lambda_s P_s^{\theta} y_s MC_{f,s}}{E_t\sum\limits_{s=t}^{\infty}(\eta\beta\left(1+\Pi\right)^{1-\theta})^{s-t}\Lambda_s P_s^{\theta} y_s}$$

In other words, the firm sets the price so that its expected value is equal to a mark up, $\frac{\theta}{\theta-1}$, over expected marginal cost. In the case of no price stickiness (i.e. $\eta = 0$), $P^*_{f,t} = \frac{\theta}{\theta-1}MC_{f,t}$. This is the standard result that under monopolistic competition firms set price as a mark up over marginal cost.

A.3 Aggregation

For tractability, full contingent claims markets are assumed. Given the ex-ante homogeneity of the households, this ensures that consumption and wealth are constant across all households. Effectively, risk averse households will insure against not being able to adjust their wage rate. Hence, $c_{h,t} = c_t \quad \forall h$ and $\Lambda_{h,t} = \Lambda_t \quad \forall h$.

It also entails that all households which are free to optimally set their wage in a given period are in exactly the same position and will choose the same wage, $W^*_{h,t} = W^*_t \quad \forall h$.

Firms which are free to set their optimal price are also all in identical positions, and so $P^*_{f,t} = P^*_t \quad \forall h$. Furthermore, I have already shown that marginal cost and the capital to labour ratio are the same across all firms.

Given that an individual firm's demand and supply must be equal, and then integrating across all firms:

$$y_{f,t} = z_t k^{\alpha}_{f,t} l^{(1-\alpha)}_{f,t} = P^{\theta}_t P^{-\theta}_{f,t} y_t$$

$$\begin{aligned}
\int_0^1 z_t k^{\alpha}_{f,t} l^{(1-\alpha)}_{f,t} df &= \int_0^1 P^{\theta}_t P^{-\theta}_{f,t} y_t df \\
\int_0^1 z_t \left(\frac{k_{f,t}}{l_{f,t}}\right)^{\alpha} l_{f,t} df &= \int_0^1 P^{\theta}_t P^{-\theta}_{f,t} y_t df
\end{aligned}$$

$$\begin{aligned}
z_t \left(\frac{k_t}{l_t}\right)^{\alpha} \int_0^1 l_{f,t} df &= y_t P^{\theta}_t \int_0^1 P^{-\theta}_{f,t} df \\
y_t &= \frac{z_t k^{\alpha}_t l^{(1-\alpha)}_t}{pd_t} \\
& \quad \textit{where} \qquad pd_t = P^{\theta}_t \int_0^1 P^{-\theta}_{f,t} df
\end{aligned}$$

In other words, aggregate output is a decreasing function of price dispersion, pd_t.

A.4 Asset Prices

The behavioral model of asset price determination is explained in section 3 above. The fundamentalist forecast in that model is premised on the fundamental asset price, q_t^*. This is defined as the asset price that would pertain to a world with rational expectations. It is the discounted sum of all future rental payments to capital:

$$q_t^* = E_t \sum_{s=t+1}^{\infty} \beta^{s-t} \frac{\lambda_s}{\lambda_t} r_s^k$$

Alternatively, this can be expressed as the discounted value of the sum of the next period rental payment and fundamental price:

$$q_t^* = E_t \beta \frac{\lambda_{t+1}}{\lambda_t} \left[r_{t+1}^k + q_{t+1}^* \right] \tag{11}$$

A.5 The Government

The government sets the nominal interest rate according to a Taylor rule which includes a response to the most recent asset mis-pricing:

$$R_{t+1}^b = r^{b^*} + \Pi_t + \zeta_\Pi \left(\Pi_t - \Pi^* \right) + \zeta_Y \ln \left(\frac{y_t}{y^*} \right) + \zeta_Q \ln \left(\frac{q_{t-1}}{q_{t-1}^*} \right)$$

The response is to the asset mis-pricing in period $t-1$ because we are in a world in which asset prices cannot easily be predicted. In rational models, policy can react to current variables, which in turn depend upon policy, because all variables are determined simultaneously. There is an implicit assumption that agents costlessly form entire response functions and costlessly and instantaneously adjust their trading volumes in response to price signals. This is inconsistent with the essence of behavioral economics. In the behavioral world, the government cannot perfectly anticipate how private agents will respond to its policy prescriptions. Hence, the simultaneous realization of monetary policy and asset prices is not within the spirit of a behavioral model. Equivalently, a solution method for such a system of equations would require the imposition of some concept of rational consistency. The Taylor rule that is most compatible with the spirit of behavioral modelling, therefore, is one in which the monetary authorities react to the mis-pricing from the previous period.

The aim in this paper is to assess whether the central bank should take account of asset prices in setting monetary policy. I will do this by comparing the welfare effects of various parameterizations of the weight on the asset price, ζ_Q.

A.6 Welfare

I use a strictly utilitarian notion of welfare, defining it as aggregate utility:

$$\begin{aligned} U_t &= \int_0^1 U_{h,t} \quad dh \\ &= E_t \sum_{s=t}^{\infty} \beta^{s-t} \left\{ \int_0^1 \ln\left(c_{h,s}\right) \quad dh - \frac{\int_0^1 l_{h,s}^{1+\phi} \quad dh}{1+\phi} \right\} \end{aligned}$$

Given the assumption of complete contingent claims markets, consumption is constant across all households. Therefore, the aggregate (or average) utility derived from consumption is just the same as the utility of consumption for any individual household. However, given price stickiness, firms employ different amounts of labour from different households, and so the aggregate disutility of labour is not straightforwardly related to the disutility of an individual household. I calculate it as follows:

$$\begin{aligned} \frac{\int_0^1 l_{h,s}^{1+\phi} \quad dh}{1+\phi} &= \frac{\int_0^1 \left(W_t^{\gamma} W_{h,t}^{-\gamma} l_t\right)^{1+\phi} \quad dh}{1+\phi} \\ &= \frac{l_t^{1+\phi}}{1+\phi} wd_t \end{aligned}$$

where $wd_t = W_t^{\gamma(1+\phi)} \int_0^1 W_{h,t}^{-\gamma(1+\phi)} \quad dh$ is a measure of wage dispersion.

Therefore, welfare is given by:

$$U_t = E_t \sum_{s=t}^{\infty} \beta^{s-t} \left\{ \ln\left(c_s\right) - \frac{l_s^{1+\phi} wd_s}{1+\phi} \right\}$$

We can clearly see that nominal rigidities have an adverse effect on welfare. Wage dispersion directly increases the aggregate disutility of work. Price dispersion, on the other hand, indirectly reduces welfare by reducing aggregate output, and hence consumption.

B Preliminary Analysis of the Dynare sub-model

The dynare sub-model consists of:

1. A set of ‘rational’ equations which is made up of the household and firm optimizations, production constraint, market clearing conditions, monetary policy rule, welfare definition and the rational asset pricing equation 11.

2. A set of ‘behavioral’ equations that is made up of exactly the same features except:

   - the asset price, q_t^B, is set equal to its fundamental value (the asset price from the ‘rational’ equations, q_t^{*R}) plus an ‘asset price shock’ term, ε_t^q.
   - the expected future asset price, q_t^{BF}, is set equal to its fundamental value (the expected future asset price from the ‘rational’ equations, $E_t q_{t+1}^{*R}$) plus an ‘expected asset price shock’ term, ε_t^{qF}.
   - the return on capital is derived from the asset price and expected future asset price via equation 8.

   The variables in this part of the model are denoted with a B superscript. Note that the expected future asset price, q_t^{BF}, is not a rational expectation of the value that the asset price, q_t^B, will take in the next period. This is where the lack of rationality enters the model, and it permeates the ‘behavioral’ equations by biasing the expected return on capital:

$$E_t r_{t+1}^{kB} = \frac{q_t^B}{\beta} \frac{\lambda_t^R}{E_t \lambda_{t+1}^B} - q_t^{BF}$$

3. A standard productivity process.

In this way, I have two, almost distinct, sets of equations. The ‘rational’ set, along with the productivity process, constitute an independent model in which all expectations are formed rationally. These equations do not depend upon the ‘behavioral’ equations in any way, and can be solved separately. The ‘behavioral’ equations, on the other hand require the fundamental asset price and rationally expected future asset price as pre-determined inputs if they are to be solved.

The dynare sub-model can be solved to provide a set of functions which determine how that model’s variables depend on predetermined variables and the four dynare ‘shocks’: the asset price 'shock', ε_t^q; the expected asset price 'shock', ε_t^{qF}; the policy response, ε_t^{MP}; and the productivity shock, ε_t. In this way I get functions which I can iterate to find the time paths of variables in the model, once I have determined the size of the 'shocks'. These are determined by the behavioral sub-model, the Taylor rule and the productivity process.

Here, I present and briefly analyze the impulse responses to all four 'shocks' in the dynare sub-model.

Figure 6: Impulse responses to a 1% productivity shock (i.e. $\varepsilon_1 = 0.01$)

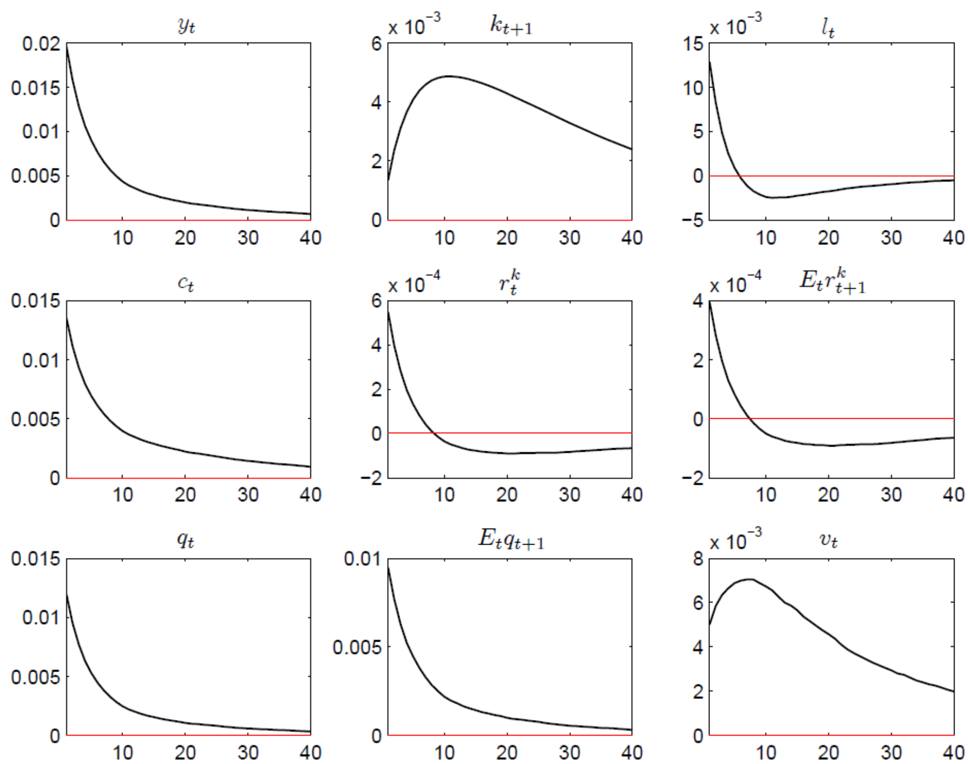

Figure 6 shows the responses of some key variables to a 1% productivity shock, with an auto-regressive component, ϱ, of 0.95. Given that there are no asset price 'shocks', the behavioral variables in dynare follow exactly the same path as their 'rational' counterparts. The direct effect of the increase in productivity increases output, y, by 1% in period 1. Output actually rises by more than this because the productivity shock also affects the employment of capital and labour. It increases the productivity of both capital, k, and labour, l, which in turn lead to increases in investment and the demand for labour. The increase in permanent income drives up consumption,

c, and reduces the supply of labour. In the periods immediately following the shock, the expansion in demand for labour outweighs the contraction in supply, but this is eventually reversed with the quantity of labour falling below its steady state value from around period 5. Discreet period utility, v, is driven mainly by changes in consumption, which are of an order of magnitude greater than the changes in labour. Initially, the return on capital and expected future return on capital are driven up by the productivity shock as the employment of labour increases rapidly, and the accumulation of capital lags behind. By period 9, however, sufficient capital has been accumulated so that the returns to capital have fallen back below their steady state level, as productivity falls back towards its steady state level. The asset price, q, shoots up in response to the productivity shock and gradually falls back to its steady state level. This is perfectly anticipated (see $E_t q_{t+1}$) since there are no other shocks after period 1.

Figure 7: Impulse responses to a 1% shock to the current asset price (i.e. $\varepsilon_1^q = 0.01$)

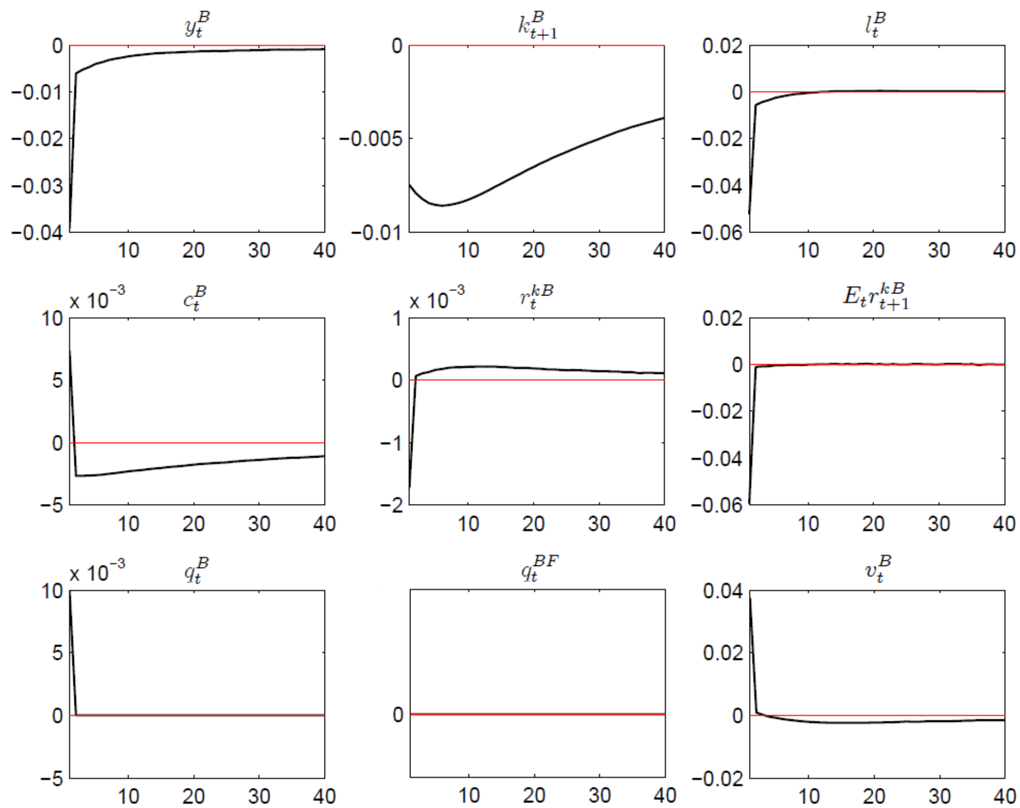

Figure 8: Impulse responses to a 1% shock to the expected future asset price (i.e. $\varepsilon_1^{qF} = 0.01$)

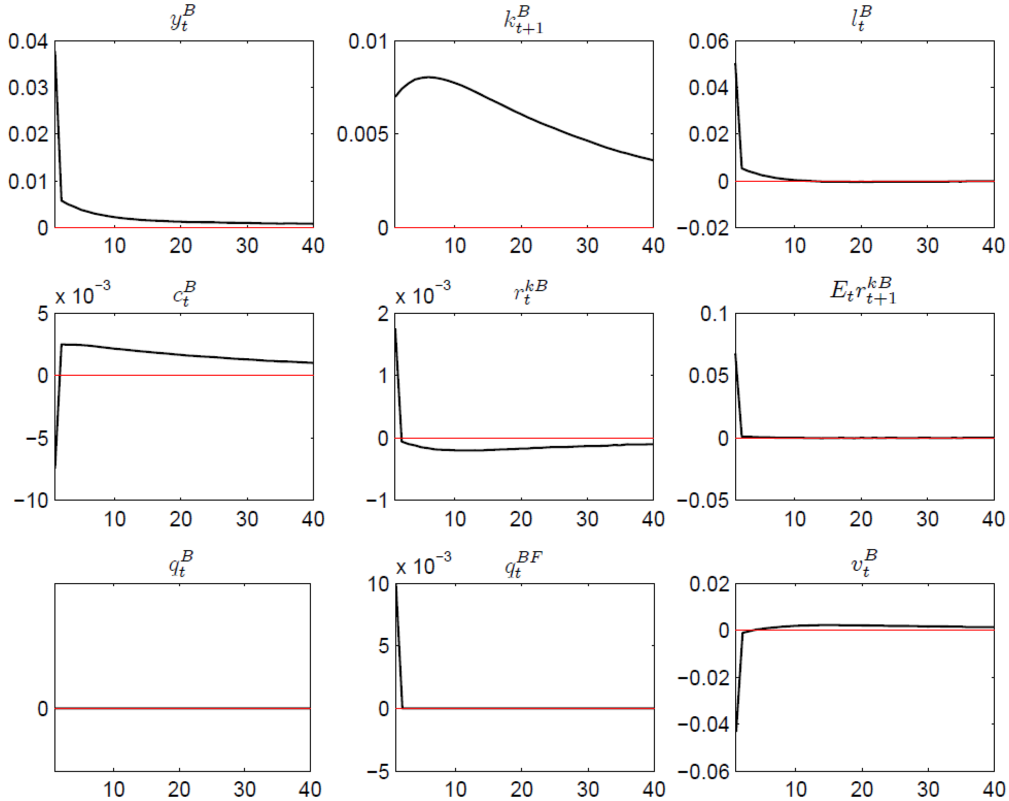

Figure 7 shows the response to a transitory 1% shock to the current asset price, whilst figure 8 shows the response to a transitory 1% shock to the future expected asset price. These represent the biases to the asset price that are determined within the behavioral model. In the dynare model, they have no direct effect on the 'rational' variables[4]. These two shocks do not occur in isolation in the full dynamic simulations of the model that we will discuss later, but for now we will consider the impulse responses separately. A positive shock to the current asset price discourages investment, boosting consumption and reducing labour supply in period 1. The under-accumulation of capital reduces output and consumption in subsequent periods.

A positive shock to the future expected asset price has the effect of increasing labour supply and reducing consumption in order to fund investment in capital, which is brought forward in anticipation of an over-pricing of capital in the next period. This drives down present period utility.

[4]However, it is worth noting that they do affect the accumulation of capital in the full model, and hence affect the initial conditions for subsequent time periods. In this way, these shocks do have an effect on future fundamentals.

Intertemporal utility increases, though it is important to note that this is ex-ante intertemporal utility. In effect, we have a shock to expectations which is a wedge between ex-ante and ex-post returns to capital and which also drives a wedge between ex-ante and ex-post intertemporal utility. In fact, by the time agents reach period 2 and recognize their mistaken beliefs in period 1, they have already accumulated extra capital which advantages them in period 2 and henceforth, though not to the extent that they had anticipated in period 1.

Figure 9: Impulse responses to a 1% shock to the Taylor rule (i.e. $\varepsilon_1^{MP} = 0.01$)

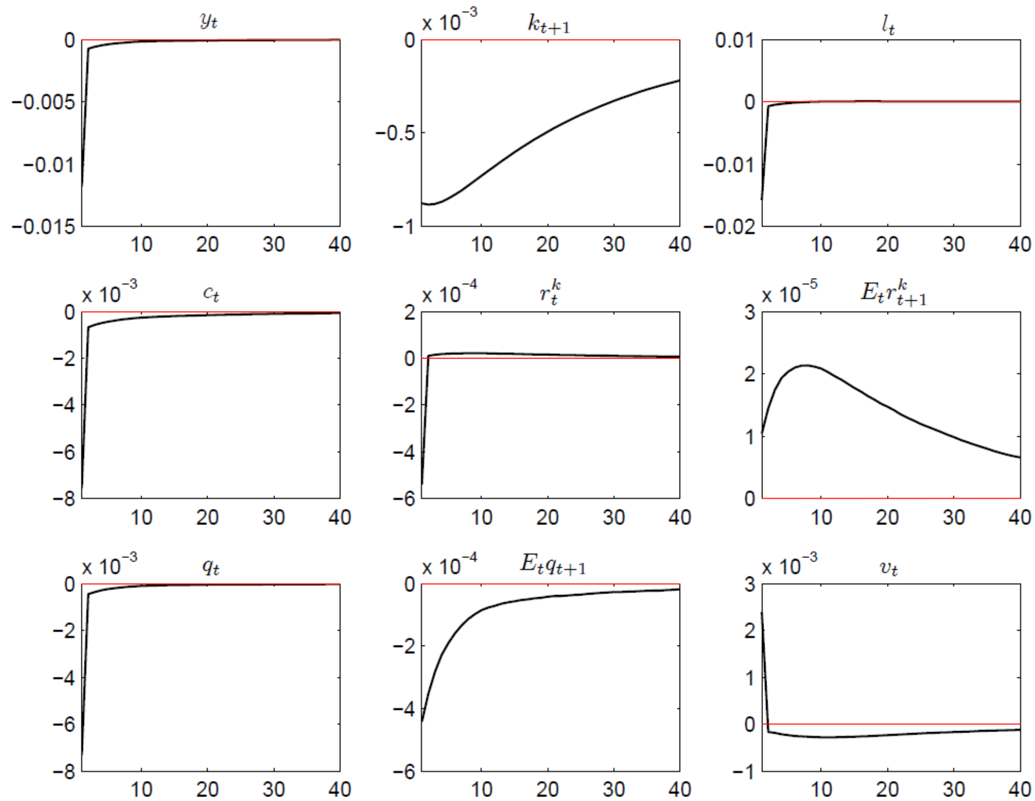

Figure 9 shows the response to a 1% transitory shock to the Taylor rule. As with the productivity shock, the monetary policy 'shock' in isolation does not cause the rational and behavioral variables in the dynare model to diverge - it affects the equivalent variables in exactly the same way. If the nominal interest rate is set above its steady state value then, in the presence of price rigidities, the real return to bond-holding increases and agents substitute out of capital and into bonds. The decrease in capital reduces the marginal productivity of labour, and hence labour falls.

A fall in output, consumption and utility ensues.

C Parameters for the Benchmark Model

Intertemporal discount factor:	β	$=$	0.99
Elasticity in the goods aggregator:	θ	$=$	7.00
Work disutility coefficient $= 1+\phi$,	ϕ	$=$	3.00
Probability Calvo fairy does not visit price setter:	η	$=$	0.67
Probability Calvo fairy does not visit wage setter:	ω	$=$	0.75
Inertia in productivity shock:	ρ	$=$	0.95
Elasticity in the labour aggregator:	γ	$=$	7.00
Depreciation rate:	δ	$=$	0.025
Coefficient in adjustment cost for investment:	ν	$=$	8.00
Share of capital in the production function:	α	$=$	0.25
Chartist rule parameter:	χ_c	$=$	$\frac{0.99}{1.99}$
Fundamentalist rule parameter:	χ_f	$=$	0.50
Uncertainty bound in fundamentalist rule:	C	$=$	0.00
Propensity to switch between forecasting rules:	υ	$=$	3.75
Taylor rule weight on inflation:	ζ_Π	$=$	2.02
Taylor rule weight on output gap:	ζ_Y	$=$	0.184
Taylor rule weight on asset mis-pricing:	ζ_Q	$=$	various
Standard deviation of productivity shock:	σ_ϵ	$=$	0.0086

D Robustness Tests

Table 2: Utility cost of various Taylor rules in the model with no switching between forecasting rules

Difference compared to benchmark model: $\upsilon = 0$		
ζ_Q	**Utility cost versus rational model** (%age permanent consumption)	**Performance relative to** $\zeta_Q = 0$ (%age bias corrected)
0.00	-0.184 %	0.0 %
0.05	-0.197 %	-6.6 %
-0.05	-0.173 %	6.2 %
0.10	-0.211 %	-14.6 %
-0.10	-0.163 %	11.8 %
0.50	-0.412 %	-123.2 %
-0.50	-0.111 %	39.9 %
1.00	-1.642 %	-790.5 %
-1.00	-0.101 %	45.5 %

Table 3: Utility cost of various Taylor rules in the model with rapid switching between forecasting rules

Difference compared to benchmark model: $\upsilon = 100$		
ζ_Q	**Utility cost versus rational model** (%age permanent consumption)	**Performance relative to $\zeta_Q = 0$** (%age bias corrected)
0.00	-0.233 %	0.0 %
0.05	-0.186 %	20.2 %
-0.05	-0.293 %	-25.6 %
0.10	-0.165 %	29.5 %
-0.10	-0.358 %	-53.4 %
0.50	-3.815 %	-1534.4 %
-0.50	-0.595 %	-154.9 %
1.00	NA %	NA %
-1.00	-0.895 %	-283.6 %

Table 4: Utility cost of various Taylor rules in the model with a 1 percent uncertainty bound around the fundamentalist forecast

Difference compared to benchmark model: $C = 0.01$		
ζ_Q	**Utility cost versus rational model** (%age permanent consumption)	**Performance relative to $\zeta_Q = 0$** (%age bias corrected)
0.00	-0.163 %	0.0 %
0.05	-0.176 %	-7.9 %
-0.05	-0.153 %	6.4 %
0.10	-0.190 %	-16.3 %
-0.10	-0.143 %	12.5 %
0.50	-0.373 %	-129.1 %
-0.50	-0.097 %	40.2 %
1.00	NA %	NA %
-1.00	-0.114 %	30.3 %

Table 5: Utility cost of various Taylor rules in the model with a 10 percent uncertainty bound around the fundamentalist forecast

Difference compared to benchmark model: $C = 0.1$		
ζ_Q	**Utility cost versus rational model** (%age permanent consumption)	**Performance relative to $\zeta_Q = 0$** (%age bias corrected)
0.00	-0.130 %	0.0 %
0.05	-0.190 %	-46.0 %
-0.05	-0.090 %	30.5 %
0.10	-0.285 %	-119.2 %
-0.10	-0.063 %	51.5 %
0.50	NA %	NA %
-0.50	-0.004 %	97.0 %
1.00	NA %	NA %
-1.00	-0.059 %	54.4 %