Arholiadau Ysgoloriaeth 2022

Scholarship Examinations 2022

Amser a ganiateir : 2 awr MATHEMATEG

Time allowed: 2 hours MATHEMATICS

*You may attempt any of the questions. Credit will be given for complete answers.*

[Assume *g* = 9.8 m.s-2]

1. The sum of the first 3 terms of an arithmetic series is 21 and the sum of the first 7 terms is 105. Find the sum of the first 11 terms.

2. Find all the solutions to the equation

3. Evaluate the definite integral

4. Find the equation of the normal to the circle

 from the point (14, -8), and the tangent to the circle at the point (7, 9).

5. Express

 in partial fractions.

6. Expand

 in ascending powers of *x*, up to the term in *x3*.

 For what values of *x* is the expansion valid?

 Find an approximation to the cube root of 1.025 using the first three terms of the expansion. How accurate is your approximation?

7. Rearrange the expression



 to yield an expression for *x*.

 Hence solve

8. The vertices *A*, *B*, *C* of a triangle have position vectors ***a***, ***b***, ***c***. Point *X* lies on the line *AB* such that *AX* : *XB* = 5 : 2. Write an expression for the position vector of *X* in terms of ***a***, ***b***, ***c***.

 Find, in terms of ***a***, ***b***, ***c***, the position vector of point *P* which divides *XC* in the ratio

*XP* : *PC* = ** : 1 - **

9. A body of mass 5 kg is suspended from a light string. The string is looped over a frictionless pulley and the other end attached to a 7 kg mass on a frictionless plane inclined at 20 degrees to the horizontal. Draw a sketch of the system. If the two masses are initially at rest, in which direction will the masses begin to move? How long will it take the vertically suspended mass to move 1 cm?

10. Solve the differential equation

 with initial condition .

11. The continuous random variable *X* has probability density function *f* defined by

 Find the parameter **, and the mean and standard deviation of *f*. Sketch the probability density function.

12. Three balls are chosen at random from a box containing 5 red balls, and 4 blue balls. Find the probability that

 a) two red balls and one blue ball are chosen (in some order)

 b) all three balls will be the same colour

13. The length of hospital stay for patients with an infectious disease comes from a normal distribution, with mean 5 days and standard deviation 2 days.

 Calculate the probability that a randomly chosen patient’s stay in hospital is between 3 days and 6 days.