MATHEMATICS IN THE PRIMARY SCHOOL

DYLAN V. JONES

ANN DOWKER

and

DELYTH LLOYD

• PRIFYSGOL CYMRU •
UNIVERSITY OF WALES
BANGOR
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Ann Dowker is University Research Lecturer in the Department of Experimental Psychology, University of Oxford, and is interested in mathematics and language development. She is joint editor of *The Development of Arithmetic Concepts and Skills* (Erlbaum, 2003), is author of the report *What Works for Children with Mathematical Difficulties?* (DfES, 2004) and of the book *Individual Difference in Arithmetic: Implications for Psychology, Neuroscience and Education* (Psychology Press, 2005).

Dylan Jones is a lecturer in the School of Education and Lifelong Learning at the University of Wales, Aberystwyth, where he co-directs the primary PGCE. His research interests include teaching and assessing mathematics within a bilingual context and he has published a number of articles in these areas.

Delyth Lloyd is a psychology graduate of the University of Oxford. Her central research interests are cultural and linguistic influences on mathematical learning. In addition to research projects with bilingual Welsh children, she has also worked on number development with language impaired children, and numerical concepts in Aboriginal Australian children.

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Contents

Foreword

'Two twos are four. You don't ask why. It just is.' Such was the response of a British Education Minister towards the end of the twentieth century to a question concerning the standards of learning mathematics. The minister was expressing the opinion and belief of many people that that is the essence of mathematics: a body of absolute facts, which need to be committed to memory and regurgitated as required.

In reality, mathematics is much more complex than that. Its concepts are abstract, they come to life through language and depend on individuals attaining an ownership of those concepts and applying them to real life. Mathematics is therefore mediated by culture and becomes alive through that culture. In this issue of *Education Transactions* we take a particular look at mathematics through Welsh eyes, drawing on both Welsh-medium and English-medium education within that particular culture.

In the first chapter Dylan Jones outlines the history of mathematics education through the medium of Welsh. He also places that history within the context of recent mathematical curriculum development in Wales side by side with its development in England. While much of the content is common, there are significant differences, in terms of emphasis and the responsibility of the teacher as a professional, that reflect historical differences in the education process within both countries.

In their chapter, Ann Dowker and Delyth Lloyd describe the results of their research on the effect of the modern method of counting in Welsh, which follows a strictly regular pattern, on children's understanding at primary level of the fundamental concept of place value. They suggest, in particular, that children, who learn the basic elements of number in Welsh rather than in English, gain an advantage and they compare this result with similar work that has studied the success of children in Pacific Rim countries, such as Japan, China and Korea.

Professor Gareth Roberts
School of Education
University of Wales, Bangor

GOOD PRACTICE IN MATHEMATICS TEACHING

DYLAN V. JONES

This chapter considers aspects of good practice in mathematics teaching in the primary school within the Welsh and Welsh-medium context. The first section summarises developments over the last fifty years with regard to the teaching of mathematics through the medium of Welsh. Following a short discussion of the wider historical context, the second section goes on to discuss elements of what is considered today as good practice and examines a number of aspects which every teacher and prospective teacher of mathematics needs to consider. In the third section, there is a discussion of the National Numeracy Strategy in England and the Framework for raising standards of numeracy in Wales. The last section offers some comments on possible future directions.

Mathematics Through the Medium of Welsh

For many minority cultures within Europe and beyond, events in Wales regarding the development of Welsh-medium education are the subject of interest and admiration. It is a story of extraordinary success and, if it is fair to describe the development of Welsh-medium education as a 'minor miracle' (Williams, 1988), then developments in the field of teaching *mathematics* through the medium of Welsh have been truly remarkable.

Until the late 1970s, the tendency within the Welsh-medium and bilingual education sector was to teach subjects such as geography, history and religious education in Welsh while science and mathematics were taught in English. There were many reasons for this, including a lack of confidence among teachers to teach these latter subjects through the medium of Welsh when they themselves had not studied them in Welsh. Others questioned the wisdom of learning these subjects in Welsh when there was little or no continuation in Welsh-medium provision at further or higher education level.

For historical and social reasons, many also believed that English was *the* language to use to 'get on in the world' and therefore doubted the wisdom of using Welsh for teaching scientific and technical subjects. Such a linguistic division of the curriculum conveyed unfortunate messages as to the 'value' of the Welsh language, and an increasing number of teachers and educationalists sought to promote its use across all aspects of the curriculum (Williams, 2003).

During the late 70s, the paucity of Welsh-medium resources was a major practical constraint. The influential Cockcroft Report, in a separate section on the teaching of mathematics through the medium of Welsh, highlighted the need for a 'suitable supply of (Welsh-medium) teaching materials' (Cockcroft, 1982:65) and for funds to be made available for this purpose. During the 80's, a number of individuals and institutions responded to this challenge and contributed to a significant widening of provision. Amongst these, the Clwyd Educational Technology Centre published the *Marthamateg/Mousematics* series for infants, and translations of the *Scottish Primary Mathematics Group* booklets were published in Glamorgan with the assistance of the Welsh Office. Resources were also developed within the teacher training institutions. For example, the mathematical magazine *Symdonics* was published at Coleg Normal in Bangor (following its initial launch by Gwynedd Education Authority) and the college developed a number of resources in co-operation with the then Language Studies Centre. Kirkman (1992) offers a detailed description of events and personalities which contributed to developments at Aberystwyth during the 70s and 80s. During, and since, this early period, the Welsh Joint Education Committee and the Centre for Educational Studies at Aberystwyth have played a major part in the process of publishing adequate teaching resources for the primary and secondary sectors. One of the most important developments in extending maths provision to the secondary sector was the publication, from 1979 on, of *Cyfres Cambria*, an original series written by a team of enthusiastic teachers and lecturers led by Dafydd Kirkman (Roberts, 1992). During the 90s, there were substantial additions to the stock of available material with the publication of a large number of textbooks and other mathematics resources, many of which were translations of popular and colourful English volumes. Although teachers within the Welsh-medium sector will never have the same range of choice as their peers in the English-medium sector, it is now fair to say that there is a substantial – if not adequate – provision of suitable resources for the study of mathematics through the medium of Welsh in our primary and secondary schools.

Approximately 12% of all primary school pupils in Wales now take their end of Key Stage 2 mathematics test in Welsh (source within the Qualifications, Curriculum and Assessment Authority for Wales [ACCAC]). At the secondary level, data on GCSE entries offers striking evidence of the growth in the number of Welsh-medium mathematics candidates. The first candidate took 'O' Level Mathematics (the forerunner of the General Certificate of Secondary Education [GCSE]) in Welsh in 1975. By 2001 the number taking their GCSE Mathematics through the medium of Welsh had increased to 3033 (WJEC, 2002). Apart from Welsh as a subject, more candidates now take the GCSE Mathematics examination through the medium of Welsh than any other subject (Jones, 1997).

It is, of course, completely natural for pupils to learn mathematics, as with any other aspect of the curriculum, through their mother tongue. In addition, it may also be argued that there are advantages to being able to consider concepts and principles through two languages. Historically, there has been much argument as to the advantages and disadvantages of bilingualism with respect to pupils' cognitive development. Contrary to the early work of those such as Saer (1923), recent international evidence (Baker, 1993) suggests a possible positive relationship between bilingualism and cognitive development. The bilingual pupil has two windows on the world, and 'deunaw' and 'un deg wyth' certainly say something in addition to what is conveyed by *eighteen.* Similarly, the terms 'uwcholwg' and 'canolrif' convey their meaning more clearly than the corresponding English terms, *plan view* and *median.*

Teaching Mathematics

(i) Recent History

Bearing in mind the current emphasis on literacy and numeracy, a layman could easily be led to believe that what happens in our primary schools today has much in common with what was to be seen half a century ago. Unlike some pupils of the 70s and 80s, pupils today, like their grandparents, have to learn their tables and perform long multiplication calculations with paper and pencil. Although elements of the educational pendulum seem to have swung back to where they were a long time ago, there are, however, important differences between what is currently considered good practice in relation to mathematics teaching, and what pertained fifty years ago. Teaching methods changed as teachers responded to both philosophical and political changes in emphasis. It may be helpful to consider the way in which some of the basic arguments developed over the period, as we seek to develop our understanding of features which are considered today to be some of the key ingredients of effective mathematics teaching (see Brown (1999), Selinger (1994) and Orton (1992) for more on the history and arguments).

Considerable tension existed between those educationalists who believed that the essence of mathematical ability was the ability to calculate correctly, and other educationalists who believed in the underlying importance of the possession of a 'number sense'. These standpoints may be further categorised into the procedural and conceptual viewpoints. Tensions also existed between those who wished to see the adoption of a progressive individualistic philosophy with its emphasis on the freedom of teachers and pupils, and others who saw education as a public process to be led by the state for the benefit of the individual and the country.

During the 1950s and 1960s the publication of reports such as *The Teaching of Mathematics in Primary Schools* (Mathematical Association, 1955), the *Plowden Report* (DES, 1967a) and the *Gittins Report* (DES, 1967b) led to an increasing emphasis on 'child-centred' ideas, and the importance of getting children to learn at their own speed by exploration and gaining experience from the world around them. These ideas influenced the *Nuffield Mathematics Project* (1964-71) and a large number of schools proceeded to introduce new aspects of maths such as logic and probability. The project led to the publication of a series of textbooks in England which emphasised the development of understanding at the expense of regular calculation practice, in the hope that the former would reduce the need for the latter. In England, however, a perception was created that some teachers followed these principles to the extreme, soon creating a backlash from those who believed that things had gone too far. Some alleged that teachers no longer taught children at all but, rather, left them in small groups to organise and teach themselves. It is important to note, however, that few children in Wales followed these more modern courses and methods, and that schools in Wales were largely traditional – particularly in the secondary sector.

Public and political concern in England regarding 'modern' methods grew during the 1970s, and a committee was established in 1978 to investigate the teaching of mathematics in schools. Four years later, *Mathematics Counts* (or the Cockcroft Report, named after the committee chairman) was published (Cockcroft, 1982). This authoritative report concluded that some of the 'modern' aspects had in fact assisted children in developing a basic understanding of mathematics, as well as improving their attitude towards the subject. It recommended that practical and investigative work, which was already happening in a number of primary schools, should have an appropriate place in the work of all primary schools and should be further developed in the secondary school.

The publication of the Cockcroft Report was an important and influential milestone. Above all, it was the Cockcroft Report which offered a lead in developments over the subsequent fifteen years. The 1988 Education Reform Act and the introduction of the National Curriculum did no more, as regards the teaching of mathematics, than give legal status to some of the main recommendations of the Cockcroft Report. The introduction of the National Numeracy Strategy in England and the parallel developments in Wales during 1999 were further important milestones which will certainly compete in terms of their historical significance with Cockcroft. These developments are discussed further in the next section.

(ii) Good Practice

One of the key paragraphs of the Cockcroft Report is paragraph 243. This is an excellent starting point for the consideration of a variety of principles which are central to effective teaching:

> **Mathematics teaching at all levels should include opportunities for**
>
> - exposition by the teacher;
> - discussion between teacher and pupils and between pupils themselves;
> - appropriate practical work;
> - consolidation and practice of fundamental skills and routines;
> - problem solving, including the application of mathematics to everyday situations;
> - investigational work.
>
> (Cockcroft, 1982: para 243)

During the pre-Cockcroft years, the tendency in the more traditional primary and secondary schools was for maths lessons to be built around periods where the teacher *explained* and demonstrated how to do the work before giving the pupils an opportunity to *practice* the skills demonstrated. At times, the new skills learned were used to *solve problems.* The paragraphs following 243 recognise the importance of these three elements (*explanation, practice* and *problem solving*). In addition, however, attention was drawn to the more 'modern' aspects such as *discussion*, *applying mathematics to everyday situations* and *investigative work*. Cockcroft's feat was to discover and describe good practice in terms of traditional methods as well as the more modern methodology.

There is nothing revolutionary in the notion that a teacher should, at times, stand in front of the class and *explain* things to pupils. Certainly, there must be periods when the teacher explains, and the challenge is to do that well. Mathematics can be a difficult subject to explain as well as to understand. Developing the ability to explain something effectively is one of the most important skills a teacher can nurture, and it is a skill which every good teacher develops and sharpens over time. With perseverance, experience and a measure of vision, the effective teacher can see how to throw light on the most difficult of concepts.

As part of the 'conceptual' philosophy, it is now recognised that one of the most important things any teacher can do is to get pupils to *understand* basic principles and to develop what Skemp (1989) called *relational understanding* rather than *instrumental understanding*. That is, you need not only to know *what* to do but to understand *why*. In performing formal subtraction sums, it was common in the past for teachers to use one of two standard methods, the Decomposition Method or the Equal Addition Method.

Although it is possible to explain the first method in terms of regrouping tens and units, it is not so easy to explain what happens in the second method, and many tended in the past to follow a rhyme which referred to 'borrowing ten and paying it back'. But where does the additional ten come from? Does 'borrowing' really occur? Can you *understand* what is happening?

Of course, such tricks can help if the only aim is to obtain correct answers fairly easily. If, however, one wishes to build a firm foundation for further mathematics, it is necessary to seek to understand the reasons behind the rules, and therefore, if a 'standard' method is used at all, teachers today almost invariably use the first method.

Practising mathematical skills was a prominent feature of the mathematical education of most people born before the 1970s, and there are aspects of mathematics where practice is necessary if tasks are to be performed quickly and accurately. Proficiency with numbers, as with many other skills, is gained through regular practice.

The need to practice number work, particularly mental work, is given prominence in the National Numeracy Strategy in England and the Framework for raising standards of numeracy in Wales (see next section). This can be done effectively by playing a game or through the more traditional question and answer method. Methods which became popular as part of the National Numeracy Strategy involve ensuring that each child has a means of responding individually to the teacher's questioning. This can be done by asking each pupil to choose the correct number card from a collection of cards in front of them, writing on a slate or small whiteboard, or responding electronically. Pupils can be asked to use their cards or whiteboards to 'show' the answer together rather than having the teacher choose an individual to answer. Such methods ensure that everyone must think about the answer and help the teacher to see who understands and who is experiencing difficulties. The teacher may also see patterns of incorrect answers which may be of help in seeking to develop the work further.

Another method of ensuring that pupils are individually engaged is the use of the computer as a question and answer tool. Integrated Learning Systems [ILS] such as *Successmaker* can, in a very short time, recognise individual pupils' strengths and weaknesses and can tailor exercises specifically for them.

As well as ensuring that pupils have an opportunity to practise their mental strategies, it is essential to ensure planning for structured progression in the way these skills are developed. The National Numeracy Strategy offers valuable guidance in this respect.

When working on calculations, it is important to encourage pupils to consider, as a starting point, whether it is possible to do the calculation mentally. If it is possible, then this, naturally, is what should be done. If the calculation is too complex to be done easily mentally, then the natural step is for the pupil to make some informal jottings in helping to arrive at the answer. For instance, in seeking to calculate 437- 251 the pupil could use the method of 'counting on ' from the smallest to the largest number, making informal notes as the process proceeds to record the successive addition of 9, 40, 100 and 37, thereby accumulating a total of 186.

This method is as valid as any other method for calculating 437-251 and it is important that pupils have the opportunity to use and share ideas about such methods. It is possible to develop an understanding of traditional 'standard' methods once pupils have a firm understanding of the central concepts.

In tandem with practising skills, it is necessary also to ensure regular and continuing opportunities to apply those skills in order to reinforce and strengthen them. The need for pupils to have opportunities to use and apply their mathematics to problems and investigations is discussed later on.

Practical experience is essential for the young child and, at times, for much older children. It is partly from practical experiences that the child develops his or her understanding of mathematics. Adding two and two is not something a small child can grasp. The concepts are totally abstract. Ask how many cows there are in a field if there are two cows at one end and two at the other, and you will see the child's face light up as you begin to talk about something which makes sense. Young children need to have experiences with all kinds of practical equipment so as to develop the network of concepts we call mathematics (Hughes, 1986; HMI, 1985). Not only, of course, is practical work of help in building mathematical understanding, but there are other times when it is absolutely essential. Measurement, for instance, is in essence a practical activity. In order to have any appreciation of what a kilogram or a litre is, one must have the opportunity to weigh and measure. The ability to use a tape measure is not developed by listening to the teacher explaining how to use it, but by getting hold of it and measuring things. A simple practical activity can also assist in understanding and remembering simple facts. If you cut three corners of a paper triangle and glue them together along a straight line, it will be easier to believe and remember that the internal angles of a triangle add up to 180 degrees.

The reference to *discussion* in the Cockcroft Report was new to many maths teachers in the early 80s. Discussion took place within other aspects of the curriculum but was not considered by many to be an important element of maths lessons. Since the days of Cockcroft, there has been an increasing emphasis on discussion and the ability to communicate effectively: 'the ability to "say what you mean and mean what you say" should be one of the outcomes of good mathematics teaching' (Cockcroft, 1982: para. 246). Following the publication of the Cockcroft Report, teachers were seen to give pupils more opportunity to discuss and develop mathematical language. It is a truism to say that it is by explaining something to another person that one often comes to understand it oneself (Brissenden, 1988). With the advent of the National Curriculum this communicative aspect was broadened and, by now, many National Curriculum test questions require pupils to explain why the answer is correct rather than merely offering an answer. Being able to communicate your ideas in correct and unambiguous language is an extremely important skill.

For young pupils, particularly, it is often not the maths which causes the difficulty, but the language used to describe it. The mathematical sentence $8 - 5 = ?$ may be read in many different ways, e.g., How many is eight take away five? How many more than five is eight? How many less than eight is five? What is the difference between five and eight? and so on. The pupil must encounter all these linguistic forms if they are to be recognised.

The ability to use mathematics for *problem solving* is essential. It is not much use knowing that $4 \times 5 = 20$ unless you know when this fact can be of practical use: 'The ability to solve problems is at the heart of mathematics' (Cockcroft, 1982: para 249). Children often have difficulty applying their mathematics to solve problems but this is an important aspect which must be fostered. Not only must one be able to read and to understand the question, but also to choose and to select the necessary mathematics before proceeding to find the answer. It is important therefore that any newly acquired mathematical skills are used in real contexts and to solve problems. A natural continuation to work on calculating the area of rectangles, for instance, would be to consider problems relating to seeding a lawn or buying a new carpet.

To schools which had not responded to the new approaches of the 1960s and 1970s, the recommendation that *investigative work* should form a natural and integrated part of work in mathematics posed a particular challenge. During the 1980s, the practice of many who were unable to respond fully to this message was to 'undertake an investigation' perhaps once a week in order to salve their conscience. But this, of course, was not in the spirit of the recommendation, and this aspect posed a challenge to many teachers throughout the 80s and 90s. In addition to finding specific opportunities to carry out investigative work, such as data collection on something of personal interest to pupils, the greatest challenge to many teachers was being able to develop an investigative teaching approach. Rather than asking what is 3 + 4 we can ask the pupil to try to find how many ways we can make 7. If we are to introduce the ratio π in an investigative manner, we can ask pupils to measure the diameter and circumference of a number of round objects (such as a bicycle wheel, plate or clock face) before encouraging them to see whether there is a relationship between these measurements for all circles. The sharpest will realise that the circumference is always approximately three times more than the diameter and it is then possible to go on to discuss this particular ratio which we know as π (there is also an opportunity here to introduce an aspect of the Cwricwlwm Cymreig by referring to the fact that it was William Jones (1675 -1749), a Welshman from Anglesey, who was the first to use the Greek letter π to denote the ratio *circumference* ÷ *diameter*).

Using and Applying Mathematics (Ma1) received special status within the National Curriculum (1988, 1991, 1995) with a separate Programme of Study allocated to it. This helped to maintain the emphasis on seeking to teach in an 'investigative manner' and the need to integrate *Using and Applying Mathematics* as a natural part of the teaching. As part of the 2000 review of the National Curriculum, there was considerable debate concerning the most appropriate way of documenting Ma1. In Wales, it was decided to retain it as a separate Programme of Study, while in England, in order to try to convey the spirit of Ma1, it became an integral part of other Programmes of Study. This is one of the chief differences between the National Curriculum 2000 Mathematics documents for England and Wales.

The opportunity to carry out all the aspects referred to in paragraph 243 of the Cockcroft Report arises very infrequently in any single lesson and this, of course, should not be the intention. The aim should be to ensure that these aspects, over a period of time, have their due place in our maths lessons. The recommendations are as relevant today as they were in the early 1980s.

The Numeracy Strategy and Framework

Since 1989 the National Curriculum has set out what teachers should teach to pupils between 5 and 16 in England and Wales and since 2000 the requirements for mathematics have been slightly different in Wales from those in England. In 1999, as part of the Government's attempt to raise achievement standards in mathematics, the National Numeracy Strategy [NNS] was implemented in England and, in Wales, *Raising Standards of Numeracy in Primary Schools: A Framework for action in Wales* (Welsh Office, 1999) was published. For the first time, an attempt was made to ensure that every primary teacher in England adopted specific teaching methods. In Wales, although there was not the same degree of prescription, there were similar messages regarding teaching methods which 'worked'.

As part of the NNS a systematic and standardised in-service training programme was developed, with detailed planning guidelines for the teaching of pupils from Reception to Year 6 (DfEE, 1999). Teachers were expected to use the 'three part lesson' template as a means of securing appropriate emphasis on the central themes of the Strategy - direct teaching and calculation work, including mental work.

The first part of the *Framework* for raising standards of numeracy in Wales places the responsibility for developing numeracy strategies in the hands of the Local Education Authorities. The second part of the *Framework* presents evidence by the Office of Her Majesty's Chief Inspector of Schools (OHMCI) regarding what characterises effective teaching of numeracy.

Although there were many similarities between the main messages of the Strategy in England and the *Framework* in Wales in relation to teaching methods, there was a fundamental difference in the way schools were expected to respond to these messages. While the Strategy in England was based on central prescription, the *Framework* for Wales aimed to promote professional co-operation as part of the process of responding to local contexts and needs.

Askew et al express the historical significance of introducing the National Numeracy Strategy in England:

> The introduction of the mathematics National Curriculum for England and Wales in 1989 was undoubtedly the most significant statutory intervention in primary school mathematics for over a hundred years. Nevertheless, the arrival of the National Numeracy Strategy into English Primary Schools in 1999 will almost certainly have had a greater impact. (Askew et al, 2001: 5)

There is no doubt that the introduction of the National Numeracy Strategy in England and the *Framework* in Wales were significant historical milestones, and understanding a little of what led to their implementation can make it easier to appreciate what they sought to achieve (see Jones (2002) for a dertailed comparison and discussion).

During the late 1980s, evidence emerged which suggested that there was a 'problem' with the way maths was taught in England (Harris et al, 1997; Reynolds and Farrell, 1996; Reynolds and Muijs, 1999). Evidence suggested that pupils in other countries, some of which were our international competitors (such as Taiwan), fared better in mathematics than pupils in England (it would be fair here to include pupils from Wales). There was also research evidence which suggested a positive correlation between particular teaching methods and higher standards of achievement. Professional evidence suggested there was room to develop teaching approaches in many primary schools. Most particularly, there was evidence of frequent over-dependence on commercial maths schemes in some schools with pupils often working on individualised tasks (i.e. doing their own work at their own speed) with very little formal, direct teacher input. In a report on teaching numeracy in three Education Authorities in England, Ofsted, the inspection body for schools in England, summarised their comments as follows:

> in the best lessons there was usually a higher proportion of time spent teaching the whole class together

in contrast with:

> poorer work which suffered from a distinct, common organisational weakness, notably a debilitating over-use of individual work, and to a lesser extent, group work. (Ofsted, 1997)

In January 1998 a report was published on the work of the National Numeracy Project in England. In this project, 520 schools were encouraged to use, among other strategies, more whole class teaching methods. According to the report, evidence suggested a link between effective teaching and whole class teaching.

> Inspection evidence and the experience of the National Numeracy Project point to an association between more successful teaching of numeracy and a higher proportion of whole class teaching. (DfEE, 1998:19)

With evidence of rising standards in those schools which were part of the Numeracy Project, all schools in England were asked to adopt the National Numeracy Strategy as from September 1999 (DfEE, 1999).

In Wales, the Welsh Office decided to adopt a less prescriptive approach than that offered by the NNS in England. In the last educational document to be published under the Welsh Office seal, *Raising Standards of Numeracy in Primary Schools: A Framework for action in Wales* (Welsh Office and OHMCI, 1999) outlined the need to raise standards through co-operation rather than compulsion. As in England, however, the need to develop pupils' mental maths skills was emphasised, as was the importance of direct teaching. Whilst the NNS in England emphasised the importance of 'whole class teaching' it is significant that the *Framework* for Wales referred to 'direct teaching'. Since there are proportionately many more small schools in Wales than in England there is also a higher proportion of classes in Wales which include pupils of 7 to 11 years of age. In such situations, it is often not as practical to 'teach the whole class' although it is possible to teach individuals or groups 'directly'.

The *Framework* for Wales noted that successful strategies for developing numeracy skills included:

- improving the children's recall of number facts
- developing mental arithmetic skills and strategies for solving problems
- reducing reliance on inappropriate use of calculators
- placing greater emphasis on opportunities to develop numeracy across the curriculum
- a clear focus on instruction through direct teaching
  (Welsh Office and OHMCI, 1999:12)

In England, there was a renewed emphasis on the idea of the three-part lesson, i.e. a lesson with an *introduction, main part* and *conclusion* and, as part of the NNS, schools are expected to follow this general pattern for the 'daily numeracy lesson' or the *numeracy hour*. Although schools in Wales are not compelled to follow this pattern, it is apparent that the Inspectorate in Wales also believes that the pattern offers a useful model.

The quality of learning is effective when teachers:

- ensure that mathematics lessons are well structured to include clear beginnings and endings which are used to introduce topics and to revise or summarise what has been learnt
  (Welsh Office and OHMCI, 1999: 27)

The *Framework* gave Local Education Authorities in Wales, in partnership with their schools, the freedom to develop their own strategies for raising numeracy standards, and to do so in a manner which accorded with their own circumstances. Although a number of the document's recommendations echo the requirements of the Numeracy Strategy in England, the decision not to adopt it on a national level was an important symbolic step in Wales which offered further evidence that there was now a willingness, with the advent of the National Assembly, to pursue policies which were fundamentally different for Wales.

The Future

At the end of the 1980s, it became clear that governments in future would determine *what* pupils should be taught. By the end of the 1990s, there were also specific expectations regarding the *how*. Some have seen this as a threat to teachers' professionalism and autonomy. Others have welcomed much of what has been offered and many have found the teaching programmes and examples developed as part of NNS, for example, to be very useful. Many teachers over the years have spent large amounts of time 're-inventing the wheel' in preparing lessons and resources and it may be argued that offering teachers more assistance with this aspect of their work frees them to do other, possibly more productive, things. A wealth of ideas and teaching resources, as well as prepared schemes of work, is already available on the internet in English. It will be interesting to see how this provision is extended over the next few years, and to what degree similar resources will be developed in Welsh.

Since the publication of *Raising Standards of Numeracy in Primary Schools : A Framework for action in Wales* (1999) more evidence has appeared that the Assembly is ready to forge its own educational path. The Assembly set out its vision for education in Wales in *The Learning Country* (National Assembly for Wales, 2001) and statutory tests for 7 year olds have already been abolished. Although some have argued that the tests have had a positive after-effect on teaching and on standards, others argue that there was too much teaching for the tests at the expense of broader mathematical experiences. Further changes in the requirements for statutory assessment at the end of Key Stages 1 and 2 in England and Wales are inevitable. More recently, there has been consultation on the proposed new 'Foundation Stage' for pupils between 3 and 7 years of age. If the proposals recommended for this stage are adopted, there will be significant implications for the way in which 5 to 7 year olds, in particular, will be taught.

It is impossible to predict what kinds of technological developments will influence our schools during the next few years, and what their likely effect on curriculum content and the way it is delivered will be. All schools in Wales already have an interactive white board, a tool which can be used in exciting and innovative ways. Further developments clearly lie ahead.

It is difficult, however, to imagine any type of society where the ability to deal fairly confidently with numbers will not be useful. Although new technologies can offer pupils very rich and appealing experiences, it is difficult to envisage a situation where the teacher's ability to explain, to excite and to inspire pupils will not lie at the heart of the teaching and learning process.

Discussion Questions

1) To what degree were the mathematics lessons which you experienced as a pupil (i) similar to, or (ii) different from, the lessons you yourself have seen or are delivering?
   Discuss common strengths and weaknesses of some of these teaching approaches.

2) During your observation/teaching periods, to what extent were/are pupils given the opportunity to experience the variety of teaching methods recommended in the Cockcroft Report, paragraph 243? What sort of teaching methods were/are most apparent? What is the reason for this?

3) The last few years have seen a shift of emphasis from *what* to teach to *how* it is taught. Do you believe that this is a development which should be welcomed?

4) How is it possible to ensure/organise *direct* teaching in a class with a wide age range?

5) When, as part of teaching and learning mathematics, is
   (a) direct teaching
   (b) group work
   (c) individual work

most appropriate?

6) What has been the effect of abolishing end of KS1 tests/tasks? Is there a danger that mathematics standards will fall with the introduction of the Foundation Stage?

7) What are the advantages and disadvantages of end of KS2 tests on learning /teaching?

8) What opportunities are there to apply mathematical skills across the curriculum?

References

Mathematics through the Medium of Welsh

Baker, C. (1993) *Foundations of Bilingual Education*. Clevedon: Multilingual Matters.
Jones, D.V. (1997) Bilingual mathematics: development and practice in Wales. *The Curriculum Journal* 8 (3), 393-410.
Kirkman, D. (1992) Datblygiad Dysgu Mathemateg Drwy Gyfrwng y Gymraeg. In W. Gareth Evans (ed.) *Fit To Educate*. Aberystwyth: Cente for Educational Studies.
Roberts, H.G.Ff. (1992) *Pwy Sy'n Cyfrif? Darlith Wyddonol Eisteddfod Genedlaethol Aberystwyth, 1992.* Llandysul: Llys yr Eisteddfod Genedlaethol.
Saer, D.J. (1923) The effects of bilingualism on intelligence. *British Journal of Psychology* 14, 25-38.
Welsh Joint Education Committee (2002) *Statistical Report on the Summer Examinations 2002*. Cardiff: WJEC.
Williams, C. (1988) *Bilingual Education in Wales or Educating for a Bilingual Wales?* Bangor: Canolfan Astudiaethau Iaith.
Williams, I.W. (2003) Y Gymraeg mewn Addysg: Ddoe a Heddiw. In G. Roberts and C. Williams (eds.) *Addysg Gymraeg, Addysg Gymreig*. Bangor: School of Education, University of Wales, Bangor

Teaching Mathematics

Brissenden (1988) *Talking About Mathematics.* Oxford: Blackwell.
Brown, M (1999) Swings of the Pendulum. In I. Thompson (ed.) *Issues in Teaching Numeracy in Primary Schools*. Buckingham: Open University Press.
Cockcroft, W.H. (1982) *Mathematics counts.* London: HMSO.
DES (1967a) *Children and their Primary School. Report of the Central Advisory Council for Education (England)*. London: HMSO.
DES (1967b) *Primary Education in Wales. Report of the Central Advisory Council for Education (Wales).* London: HMSO.
HMI (1985) *Mathematics from 5 to 16*. London: HMSO.
Hughes, M. (1986) *Children and Number: Difficulties in Learning Mathematics.* Oxford: Blackwell.
Mathematical Association (1955) *The teaching of mathematics in Primary Schools*. London: Bell.
National Assembly for Wales (2000) *Mathematics in the National Curriculum in Wales*. Cardiff: ACCAC.
Orton, A. (1992) *Learning Mathematics : Issues, Theory & Classroom Practice (2nd edition).* London: Cassell.
Selinger, M. (ed.) (1994) *Teaching Mathematics.* London: Routledge.
Skemp, R.R. (1989) *Mathematics in the Primary School.* London: Routledge.

The Numeracy Strategy and Framework

Askew, M., Millet, A., Brown, M., Rhodes, V. and Bibby, T. (2001) Entitlement to attainment: tensions in the National Numeracy Strategy. *The Curriculum Journal* 12(1): 5-28.

DfEE (1998) *Numeracy Matters*. London: DfEE.

DfEE (1999) *The National Numeracy Strategy: Framework for teaching mathematics from Reception to Year 6*. London: DfEE.

Harris, S., Keys, W. and Fernandes, C. (1997) *Third International Mathematics Study, Second National Report, Part 1: Achievement in Mathematics and Science at Age 9 in England.* Slough: National Foundation for Educational Research.

Jones, D. V.(2002) Polisiau rhifedd ar gyfer ysgolion cynradd Cymru a Lloegr: gwahaniaethau sy'n cyfrif. *Welsh Journal of Education* 11 (2), 26-40.

National Assembly for Wales (2001) *The Learning Country.* Cardiff: National Assembly for Wales.

OfSTED (1997) *The Teaching of Number in Three Inner Urban LEA's*. London: OfSTED.

Reynolds, D. and Farrell, S. (1996) *Worlds Apart? A Review of International Surveys of Educational Achievement Involving England: OfSTED Reviews of Research Series.* London: HMSO.

Reynolds, D. and Muijs, D. (1999) The Effective Teaching of Mathematics: a review of research. *School Leadership and Management* 19 (3), 273-288.

Welsh Office and OHMCI (1999) *Raising Standards of Numeracy in Primary Schools: A Framework for Action in Wales*. Cardiff: HMSO.

LINGUISTIC INFLUENCES ON NUMERACY

ANN DOWKER and DELYTH LLOYD

There are significant international differences in arithmetical performance between children in different countries (TIMSS, 1996). Children in 'Pacific Rim' countries, such as Japan, China and Korea tend to perform particularly well in arithmetic.

There are many reasons why such differences might occur, including educational methods, home attitudes to mathematics, and economic factors. However, one of the cultural characteristics that could influence children's arithmetical development is the way in which numbers and arithmetical relationships are expressed in a language.

The possible importance of this factor has been recognized for a very long time indeed, although systematic research has not been carried out until quite recently. Locke (1690) argued that small numbers can be represented without words by showing numbers of fingers, but words are needed to keep track of larger numbers. Thus, speakers of languages without number words would be restricted to the understanding of numbers that can be represented through fingers (10 to 20 or so, depending on whether they are with someone else whose fingers can be counted together with their own). Edgeworth and Edgeworth (1798) pointed out that English speakers may be at a disadvantage compared with speakers of some other languages due to the relatively irregular English counting system.

The linguistic characteristics that may be relevant may be classified as follows:

(1) *Whether the language includes number words at all; and whether, if it does, there is an upper limit to what is counted.*

Most languages have number words at least up to 10. There are some exceptions. Some Australian Aboriginal languages, such as Aranda, have only words corresponding to "one, two, three, many". A rather larger number of languages have limits on how far one can count; for example, some of the languages of Papua New Guinea count by pointing to body parts and use the names of these body parts for their counts (Lancy, 1983; Butterworth, 1999). Thus, in the Kewa language '1' is represented by the right little finger, and '34' by the nose. The upper limit of the Kewa counting system is 68, while that of the somewhat similar Oksapmin system is 19.

The lack of a verbal counting system is likely seriously to constrain the development of arithmetic. It need not prevent it altogether: many studies of pre-verbal infants and of individuals with language impairments show that quantities and even arithmetical operations can be represented non-verbally.

If there is an upper limit on the counting sequence in a language, then this may not only interfere with arithmetic and quantity representation beyond that number, but may limit the ability to understand a key central mathematical concept, that of infinity: the fact that a quantity can in theory be increased indefinitely, without limit.

There has been very little research on the effect on mathematical understanding of growing up in a culture with no verbal counting system, or a counting system with an upper bound. Such research would seem to be extremely important, both from the point of view of understanding cultural influences on mathematics, and on understanding the extent to which language and number concepts are interrelated. Current research in Australia, led by Brian Butterworth and Robert Reeve, focuses on number concepts and skills in child speakers of Aboriginal languages with few or no number words.

(2) *The base of the counting system*

The counting system generally used today is base 10. In the past, bases have included base 5, base 20 and base 60. Base 12 has its survivals in terms such as 'a dozen' and 'a gross', and in the tendency for some items such as eggs to be sold in 'dozens' or 'half-dozens'. The French verbal counting system uses 20 as a reference point. For example, whereas 70 is 'soixante-dix' (sixty-ten), 80 is 'quatre-vingts' (four-twenties), and 90 is 'quatre-vingts-dix' (four-twenties-ten).

Some current or recent currency and measurement systems use bases other than 10, including more than one base within the same system. Until 1971, the British currency system operated on the principle of 12 pennies in a shilling and 20 shillings in a pound. The Imperial measurement systems are typically not decimal (e.g. 12 inches in a foot; 3 feet in a yard; 16 ounces in a pound; 14 pounds in a stone). These measurement systems are still used in many parts of the world. Britain changed officially to the metric system in the 1970s, and children learn only the metric system in schools. However, there is still frequent use of the older measurement systems in everyday practical contexts and many parents still think predominantly in terms of the older systems. This arises possibly because of the division between within-school and out-of-school measurement systems. Clayton (1988) found that children tended to use the metric system for exact measurement, and the older systems for estimates. It is not yet clear whether this will change, now that many of the first generation to learn the metric system in school have become parents themselves.

Once again, there has been little research into the effect of using different bases. It would be interesting to know whether the use of a base 10 system is intrinsically easier to acquire and use than other systems, due to the fact that we have 10 fingers. Such a study would be difficult, however, as there are few, if any, groups who use a system that is *exclusively* in a base other than 10. However, it would be much easier to carry out a study of the effects of exposure only to base 10 versus exposure to other bases as well. This could be done, for example, by comparing individuals in countries that use only the metric system of measurement with those in countries with other measurement systems that use bases other than 10.

(3) *Whether there is a written number system*

Some languages still do not have a written form, including for the number system. The extent to which the existence of a written number system affects arithmetical understanding is not clear. A written system not only makes it possible to keep permanent records but reduces the load on memory during arithmetical calculations. On the other hand, many people now believe that excessive concentration on written calculation at an early age may interfere with the development of deep mathematical understanding and flexible strategy use, and that it is better to begin with mental calculation.

It is difficult to study the effects of the existence of a written number system, due to confounding factors such as exposure to schooling, and literacy within the culture, etc.

(4) *Whether, if there is a written system, it is 'regular' in terms of giving a clear and consistent representation of the base system (usually base 10) used in the language.*

The Arabic number system that is almost universally used in writing today is highly regular, representing place value in a consistent fashion. However, some written number systems are irregular, including those used by the Romans. Although there were separate symbols for units (I), tens (X), hundreds (C) and thousands (M), there were also special symbols for numbers involving multiples of 5: V for five; L for 50; D for 500. Moreover, numerals sometimes represented addition to a salient number (VI as 5 + 1, i.e. 6; XIII as 10 + 3, i.e. 13) and sometimes subtraction from a salient number (IV as one less than 5, i.e. 4; IX as one less than 10, i.e. 9).

Calculation would appear to be far more difficult in Roman numerals than Arabic numerals (Flegg, 1989; Butterworth, 1999). Some historians have argued that the use of Roman numerals was a contributory factor to the low level of arithmetical skills in the Middle Ages, and that arithmetical skills improved when Arabic numerals came into greater use (Flegg, 1989), although it is difficult to establish the exact level of arithmetical skills at different periods in history.

(5) *The regularity of the spoken number system: the degree to which it gives a clear and consistent representation of the base system (usually base 10) used in the language.*

(6) *The degree and consistency of conformity between the spoken and the written number system.*

Since most languages currently use the highly regular Arabic written number system, there is in fact little distinction between (5) and (6), and they are generally not distinguished in research. It is, however, important to bear in mind that the degree of regularity of an oral counting system could be important *either* because the base system that it uses is made explicit, *or* because the oral counting system is consistent with the written counting system, and that the two need not be exactly the same.

The effect of regularity of the counting system has been the subject of a considerable amount of research. East Asian languages such as Chinese, Japanese and Korean, have very regular oral counting systems according to both of the above criteria. They correspond closely to the written number system, and they make the relationship between units, tens and higher powers of ten very explicit. For example, in these languages the number word for 12 is the equivalent of 'ten-two', and the number word for 23 is the equivalent of 'two-ten-three'. Irregular number words such as the English 'twelve' and 'twenty' do not occur in these languages.

It is sometimes suggested that the relative regularity of Asian counting systems is a major factor that contributes to the superior performance of Pacific Rim children in most aspects of arithmetic. Learning number names may be easier in systems where new numbers may be inferred rather than having to be learned by rote. Therefore a regular counting system would make it easier for young children to count to higher numbers at an earlier stage than those who have to cope with a more irregular counting system, and this might give them a head start in manipulating numbers. One might also expect that the concept of place value would be easier to comprehend and use in a regular counting system. Essentially, place value *means* the representation of the base ten system by written symbols. One might hypothesise that it is correspondingly difficult for English-speaking children to acquire the concept of place value.

Indeed, there is considerable evidence that speakers of Asian languages perform better than speakers of less regular counting sequences, both in learning the counting sequence and in learning to represent tens and units.

Miller, Smith, Zhu and Zhang (1995) studied counting in Chinese and American 4- and 5-year-olds. The two groups performed similarly in learning to count up to 12, but the Chinese children were about a year ahead of the American children in the further development and counting of higher numbers.

As regards the development of the understanding of tens and units, Irene Miura and her colleagues studied 6-year-old children of different nationalities (Miura, Kim, Chang and Okamoto, 1988; Miura, Okamoto, Kim, Steere and Fayol, 1993; Miura and Okamoto, 2003). These included three groups who used regular counting systems - Japanese, Korean and Chinese - and three groups who used less regular counting systems - American, French and Swedish. The tasks involved representation of two-digit numbers with base ten blocks (unit blocks and tens blocks; the latter being blocks with ten segments shown on them). None of the children had previous experience with base ten blocks. The users of regular counting systems were far more likely than the users of irregular counting systems to represent the tens and units by means of the blocks, typically by representing 42 by four tens blocks and two unit blocks. The American, French and Swedish children tended to attempt to represent the numbers as collections of units, such as by representing the number 42 as 42 unit blocks.

Similar results have been obtained with Korean children (Song and Ginsburg, 1988; Fuson and Kwon, 1992).

However, it is difficult to draw firm conclusions on this matter, because there are so many other cultural and educational differences between Asian and Western children (Towse and Saxton, 1998).

The Welsh language and numeracy

The Welsh language can offer important insights here. Historically, there has been more than one Welsh counting system, and an older system is still occasionally used in contexts such as dates or telling the time (Roberts, 2000). However the main counting system used for school mathematics, like the counting systems used in Pacific Rim countries, is completely regular (Roberts, 2000). The number words are easily constructed by knowing the numbers 1 to 10 and the rule for combining them. For example, eleven in Welsh is *un deg un* (one ten one), twelve is *un deg dau* (one ten two), and twenty two is *dau ddeg dau* (two ten two).

Wales provides an unusual opportunity for research on linguistic influences on mathematics, since it is a region in which languages with both regular and irregular counting systems are used. In Wales, children receive either English- or Welsh-medium schooling within the same country, educational system, curriculum, and cultural environment. In some cases, Welsh- and English-medium education even takes place in different streams in the same school. Children whose parental language is English may still receive their education from age 4 entirely in Welsh. This makes it possible to compare groups with varying levels of exposure to the regular and irregular number systems: (i) children whose first language is Welsh, have a Welsh home environment and a Welsh-language schooling; (ii) children who receive a Welsh-language schooling, but have English-speaking parents and home environment, and for whom English is their first language; (iii) children whose first language is English and who receive an English-medium education. Any extraneous cultural or educational differences between these groups will certainly be far less than those between, for example, English and Chinese children.

Moreover, speakers of Asian languages might be advantaged not just because their counting systems are regular, but because their number words are short and take up relatively little space in working memory. Welsh number words and phrases are actually longer than their English counterparts, and digit span in Welsh is correspondingly shorter than in English (Ellis and Hennelly, 1980), so that any advantage of the Welsh counting system is far more likely to be due to its regularity.

There is indeed some evidence that children in Welsh-medium schools show better mathematics performance in government school performance tables (based mainly on SATS and GCSE results) than those in English-medium schools (Bellin, Farrell, Higgs and White, 1996; Reynolds and Bellin, 1996; Reynolds, Farrell and Bellin, 2002). They obtain better results in some other subjects as well, and the issue is complicated by some social class differences between English- and Welsh-medium schools. However, even when researchers control for social class, pupils in Welsh-medium schools still seem to obtain rather better results than those in English-medium schools (Bellin et al, 1996; Reynolds et al, 2002).

There have, however, been few studies that have focused on how Welsh- and English-speaking children perform on specific aspects of arithmetic. Maclean and Whitburn (1996) studied children in their first year of school, and found that those in Welsh-medium schools performed better than those in English-medium schools on certain numerical measures. In particular, they could count higher. Comprehension and use of multi-digit numbers, which might be predicted to be particularly facilitated by the Welsh number system, was hard to assess in their study, as most of the children were 6 years old or under, and had not been much exposed to oral and written representations of tens and units.

We have carried out a study investigating the performance of numerical tasks by Welsh children who had just begun dealing with such representations (6 year-olds) and those who had greater experience (8 year-olds).

A total of sixty children drawn from three primary state schools in south Wales (in areas of similar socio-economic status) participated in the testing. There were 10 six year-olds and 10 eight year-olds from each school. One was a Welsh-medium school in a predominantly Welsh-speaking valley. Welsh was the first language for the children; they all received a Welsh-medium education, and also came from Welsh-speaking homes. This school is henceforth referred to as WW. The second was a Welsh-medium school in a predominantly English-speaking area of Wales (henceforth referred to as WE). The children attending school WE spoke English as a first language, but received education entirely through the medium of Welsh. School 3 was an English-medium school in the same town as school WE. Although situated in the same education system, country and cultural environment as the Welsh-medium schools, school 3 is an English-medium school in an English-speaking area (henceforth referred to as EE). The English-educated children of school EE were compared to those educated through the Welsh-medium in schools WW and WE.

The schools were similar in their social class intake. All were in relatively middle-class catchment areas. For example, they included a similarly small proportion of children who were eligible for free school meals.

The children were given three standardized tests: the British Abilities Scales (BAS) Basic Number Skills test, which measures written calculation; the WISC Arithmetic subtest which measures mental arithmetical reasoning, especially word problem solving; and the WISC Block Design subtest which measures nonverbal reasoning (WISC, 1991). They were also given a Number Comparison task, based on that used by Donlan and Gourlay (1999).

In the Number Comparison task, 24 pairs of two-digit numbers were presented to children in a flip booklet. There were 3 types of number pairs: transparent, misleading and reversible. Transparent word pairs required judgement between numerals that either had different number of tens but the same number of units (decade comparisons; e.g. 73 and 63) or contained repeated digits, e.g. 11 and 99. In misleading number pairs the smallest number always contained a digit that was larger than the sum of the digits in the target item, e.g. 51 and 47, 19 and 21. Reversible pairs included, for example, 76 and 67, 25 and 52.

Twenty-four pairs of numbers were presented in all, eight of each type, in a random order. All participants were required to read each pair of numbers aloud before pointing to which was the bigger.

Hesitations (where children nearly pointed at the incorrect answer, and then changed to the correct one at the last second), *Misreadings*, and *Incorrect Answers* were recorded. A *Comparison Error* score indicated overall performance on the task and was calculated by finding the total number of Hesitations, Misreadings and Incorrect Answers.

Children were tested individually. For schools WW and WE all communication was in Welsh, and for school EE it was in English.

It was important to check that the groups did not differ in overall ability. Table 1 shows children's performance on the standardized tests, according to school and age group. The WISC Block Design subtest was used as a measure of nonverbal reasoning. A two-factor analysis of variance, with School and Age as factors, showed that the scaled score on this test did not differ significantly either between schools or between age groups.

Table 1

Children's scores on standardized tests

		WISC Block design		WISC Arithmetic		Basic number skills		Number of pupils
School[1]		Mean	s.d.	Mean	s.d.	Mean	s.d.	
WW	Age 6	11.5	3.44	11.3	4.27	116.0	16.96	10
	Age 8	9.4	2.88	10.4	2.67	110.4	10.10	10
	Total	10.45	3.27	10.85	3.50	113.2	13.90	20
WE	Age 6	11.6	2.55	10.8	3.39	115.4	10.65	10
	Age 8	10.0	2.53	10.3	1.49	105.6	10.89	10
	Total	19.8	2.61	10.55	2.56	110.5	11.63	20
EE	Age 6	10.5	2.68	9.2	3.46	111.7	10.35	10
	Age 8	9.9	3.00	10.7	1.40	105.9	7.78	10
	Total	10.2	2.78	9.95	2.68	108.8	9.39	20
Overall total		10.48	2.86	10.45	2.92	110.83	11.72	60

[1]WW: Welsh first language children in a Welsh medium school
WE: English first language children in a Welsh medium school
EE: English first language children in an English medium school

The schools also turned out not to differ in terms of overall arithmetical reasoning or calculation ability. A two-factor analysis of variance with School and Age as factors was also applied to the WISC Arithmetic and BAS Number Skills scores. No statistically significant differences were found between schools or age groups on the scaled scores on either test. This suggests that the counting system on its own does not appear to have an impact on global arithmetical ability in otherwise culturally similar groups.

However, it is questionable whether there *is* such a thing as global arithmetical ability! The evidence suggests that arithmetical ability is made up of many components, which correlate imperfectly with one another, and may be selectively impaired or enhanced (Dowker, 1998). Might the nature of the counting system have an effect on some more specific aspects of arithmetic? It appeared that this was indeed the case as there were group differences in more specific areas of arithmetical ability, notably in the ability to read and judge number pairs, as shown by the Number Comparison task. Although almost all children performed at ceiling level for correct answers, they varied significantly in the misreadings of numbers and hesitations in judgement.

The Comparison Error score was constructed as the most complete measure of the all-round performance of children in the Number Comparison task, since it combines misreadings, hesitations and incorrect judgements. These data presented in table 2 show the frequency of different types of error, and the composite Comparison Error score, according to school and age group.

Table 2

Mean error scores

		Misreadings		Hesitations		Incorrect answers		Comparison error score (total)		Number of pupils
School[1]		Mean	s.d.	Mean	s.d.	Mean	s.d.	Mean	s.d.	
WW	Age 6	0.3	0.48	1.0	0.94	0.9	1.67	2.2	2.15	10
	Age 8	0	0	0.4	0.70	0	0	0.4	0.70	10
	Total	0.15	3.67	0.7	0.86	0.45	1.23	1.3	1.81	20
WE	Age 6	1.4	1.70	1.9	1.37	0.9	0.88	4.2	2.20	10
	Age 8	0.5	0.85	0	0	0.1	0.32	0.6	0.97	10
	Total	0.95	0.24	0.95	1.40	0.5	0.76	2.4	2.47	20
EE	Age 6	2.6	2.17	1.5	1.80	0.7	1.64	4.8	3.77	10
	Age 8	1.1	1.2	1.6	0.71	0.1	0.32	2.8	2.53	10
	Total	1.85	1.87	1.55	1.40	0.4	1.90	3.8	3.29	20
Overall total		0.98	1.40	1.07	1.26	0.45	1.06	2.50	2.75	60

[1]WW: Welsh first language children in a Welsh medium school
WE: English first language children in a Welsh medium school
EE: English first language children in an English medium school

The composite Comparison Error score was found to show highly significant differences in a two-way analysis of variance between schools ($F(2,59)= 5.99$, $p<0.01$), and between age groups ($F(1,59)=17.4$, $p<0.001$). Children at school WW performed better than those at school WE, who in turn performed better than those at school EE; and older children performed better than younger children.

Older children also performed significantly better than younger children on each of the individual components of the Comparison Error score: misreadings ($F(1,59) = 10.1$; $p < 0.01$); hesitations ($F(1,59) = 7.0$; $p < 0.05$); and incorrect responses ($F(1,59) = 11.19$; $p < 0.01$). There were significant differences between schools on misreadings ($F(2,59) = 10.11$; $p < 0.01$); near-significant differences on hesitations ($F(2,59) = 3.09$; $p = 0.06$); but differences between schools were not significant for incorrect responses, perhaps due to ceiling effects among the older children.

In order to refine the investigation of the effect of the Welsh language, a 'Welshness' scale was constructed to compare schools in a regression analysis. 'Welshness' scores of 0, 1 and 2 were allocated to EE, WE and WW respectively. The analysis revealed that Welshness does not significantly affect BAS number skills scores or WISC Arithmetic scores. However, there was a highly significant effect of Welshness (i.e. school) on Comparison Error scores (slope = -0.380, $t = -4.057$, d.f. = 2.58, $p<0.001$).

Thus, it appears that there are some differences between the mathematics skills of children who learn mathematics in Welsh and English. These were revealed in specific areas of children's performance, but not on more general arithmetical performance as measured by WISC Arithmetic and BAS Number Skills tests. Welsh-speaking children find it easier than English-speaking children to read and compare two-digit numbers, suggesting that they are better at using the principles of place value.

It is important to emphasize that, in contrast with most studies of linguistic effects on mathematics, cultural and educational differences were not strong confounding variables in this study, which means that linguistic differences were more likely to have been causal factors.

The advantages of speaking Welsh appeared to hold, even if it was not the child's first or only language. Children who had no pre-school knowledge of a regular counting system, but who attended a Welsh-medium primary school, appeared to benefit from the introduction of this knowledge during their primary education. Children in the WE group gained to the extent that they outperformed monolingual English children in number reading and number comparisons when tested in what is their second language after just 2 years of Welsh education, at age 6, supporting somewhat similar findings for Korean-English bilingual children (Song and Ginsburg, 1988). The simplicity of the number naming process in Welsh, as in Korean, may encourage easier acquisition and earlier competence, leading to better test performances. Miller et al (1995) found linguistic differences in counting ability between English and Chinese speaking 3- to 5-year-olds; the present study suggests that differences in number skills continue beyond the preschool years, well into primary education.

The study also suggests that the effects of language on mathematics, though they are important, are quite specific. Children who use a regular counting system are not better at all aspects of calculation. The globally better performance of Pacific Rim children may be attributable not only to linguistic factors, but to other cultural factors, such as attitudes to mathematics, and amount of time devoted to it in the school curriculum. But the study *does* suggest that linguistic factors do influence the ability to use place value in reading, comparing and manipulating two-digit numbers.

Further studies, involving larger numbers of children, and a wider age range, would be desirable in order to confirm the generality of these findings. For example, although the schools were very similar in all respects except for medium of instruction (e.g. they were in similar catchment areas and followed the same curriculum), it is not possible to rule out effects of specific teachers in a sample of this size. If the findings were replicated in more schools, one could be more confident that the differences were due specifically to language.

It should be noted that very few children learn *only* Welsh. Children who attend Welsh-medium schools do also learn English; so, ultimately, they will learn two counting systems. In fact most learn three counting systems. In due course they become exposed not only to the contemporary regular Welsh decimal counting system, but also to an older predominantly vigesimal (base twenty) system, which, though it is not used in schools or in mathematics instruction, still has some place in the culture (Roberts, 2000). Thus, Welsh-speaking children start out by being exposed to a simple, regular counting system, but eventually are exposed to the complexity of multiple counting systems. The advantages and disadvantages of such exposure to multiple systems remain to be studied.

Bibliography

Bellin, W., Farrell, S., Higgs, G. and White, S. (1996). A strategy for using census information in comparison of school performance. *Welsh Journal of Education*, *5*, 3-25

Butterworth, B. (1999). *The Mathematical Brain*. Oxford: Oxford University Press.

Clayton, J. (1988). Estimation in schools. *British Society for Research into Learning Mathematics: Proceedings of the Weekend Conference, Warwick, May 14th-15th, 1988* (pp. 9-10).

Donlan, C. and Gourlay, S. (1999). The importance of non-verbal skills in the acquisition of place value knowledge: evidence from normally developing and language-impaired children. *British Journal of Developmental Psychology*, 17, 1-19.

Dowker, A. (1998). Individual differences in normal arithmetical development. In C. Donlan (ed.) *The Development of Mathematical Skills*. Hove: Psychology Press.

Edgeworth, M. and Edgeworth, R.L. (1798). *Practical Education*. London.

Ellis, N.C. and Hennelly, R.A. (1980). A bilingual word length effect: implications for intelligence testing and the relative ease of mental calculation in Welsh and English. *British Journal of Psychology*, 71, 43-52.

Flegg, G. (ed.) (1989). *Numbers Through the Ages*. London: Macmillan.

Fuson, K. and Kwon, Y. (1992). Korean children's understanding of multi-digit addition and subtraction. *Child Development*, *63*, 491-506.

Lancy, D. (1983). *Cross-Cultural Studies in Cognition and Mathematics*. New York: Academic Press.

Locke, J. (1690). *An Essay Concerning Human Understanding*. London.

Maclean, M. and Whitburn, J. (1996). Number name systems and children's early number knowledge: a comparison of Welsh and English speakers. Paper presented at XVIth Biennial ISSBD Conference, Quebec City, August 1996.

Miller, K.F., Smith, C.M., Zhu, J. and Zhang, H. (1995). Preschool origins of cross-national differences in mathematical competence: the role of number-naming systems. *Psychological Science*, 6, 56-60.

Miura, I. and Okamoto, Y. (2003). Language supports for mathematics understanding and performance. In A. Baroody and A. Dowker (eds.) *The Development of Arithmetic Concepts and Skills*. Mahwah, N.J.: Erlbaum (pp. 229-242).

Miura, I., Kim, C., Chang, C.M and Okamoto, Y. (1988). Effects of language characteristics on children's cognitive representations of number: cross-cultural comparisons. *Child Development*, 59, 1445-1450.

Miura, I., Okamoto, Y., Kim. C., Steere, M. and Fayol, M. (1993). First graders' cognitive representation of number and understanding of place value: cross-cultural comparisons - France, Japan, Korea, Sweden and the United States. *Journal of Educational Psychology*, 85, 24-30.

Reynolds, D. and Bellin, W. (1996). Welsh-medium schools: why they are better. *Agenda, Summer 1996*. Cardiff: The Institute of Welsh Affairs.

Reynolds, D., Farrell, S. and Bellin, W. (2002). Europe: the United Kingdom. In D. Creemer, B. Stringfield, S. Teddlie and G. Schaffer (eds.) *World Class Schools: International Perspectives on School Effectiveness*. London: Routledge (pp. 229-252).

Roberts, G. (2000). Bilingualism and number in Wales. *International Journal of Bilingual Education and Bilingualism*, 3, 44-56.

Song, M. and Ginsburg, H.P. (1988). The effect of the Korean number system on children's counting: a natural experiment in numerical bilingualism. *International Journal of Psychology*, 23, 275-302.

TIMSS (1996). *Highlights of Results from TIMSS: Third International and Science Study*. Chestnut Hill, MA: TIMSS International Study Centre.

Towse, J. and Saxton, M. (1998). Mathematics across national boundaries. In C. Donlan (ed.) *The Development of Mathematical Skills*. Hove: Psychology Press (pp. 129-150).

WISC (1991). *Wechsler Intelligence Scale for Children* (3rd edition). London: Psychological Corporation.