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Abstract

Economic agents with hyperbolic discount functions display time inconsistent preferences. In this paper, I show that for such agents fixed nominal wage contracts may represent a welfare enhancing commitment mechanism.

Key words: Nominal rigidities, Hyperbolic discounting

1 Introduction

The existence of nominal wage rigidities is a fact accepted by most macroeconomic modelers. Various explanations of the source of theses rigidities exist within the literature. These include money illusion, menu costs and information asymmetries. However, all of these explanations remain controversial.

In this paper, I pursue an alternative explanation of nominal wage rigidity. I consider under what circumstances fixed nominal wage contracts represent an optimal commitment mechanism for hyperbolic discounters.

The discount rate that an exponential discounter uses to compare rewards which are realized at different points in time depends only upon the length of time between those realizations. For hyperbolic discounters, however, it also depends upon the proximity of those rewards to the present time period. They apply different discount rates when calculating today's value of a reward which will be received in a year's time as compared with calculating the value in 2015 of a reward which will be received in 2016. The ubiquity of this behavior, where

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the salience of the presence dominates decision making, is well supported by empirical evidence.

It is well established that the time inconsistency inherent in hyperbolic discounting means that commitment mechanisms can be welfare enhancing [1] (e.g. Laibson 1997). A rational hyperbolic agent will recognize the time inconsistencies in its own behavior. The agent at time zero (self 0) will want to implement a plan for the entire time horizon which it knows will not be considered optimal by itself in time period 1 (self 1), and will therefore not be implemented. Self 0 will, therefore, have an incentive to take advantage of mechanisms which constrain the behavior of self 1, and oblige self 1 to follow self 0's plan.

The salience of utility at time period 1 to self 1 will encourage the agent to consume more and work less than would be perceived as optimal by self 0. From the perspective of self 0 the optimal solution is to tie future selves into wage contracts which obligate a higher labour supply than would be chosen by those future selves.

A recent paper by Graham and Snower (2008) shows how nominal rigidities lead to higher labour supply in a model with hyperbolic discounting and positive inflation. Therefore, a fixed nominal wage contract may be optimal if it enables households to commit their future selves to a higher labour supply.

2 Inter-temporal Allocation

The common assumption of mainstream economic models is that agents will allocate their resources across time so that the marginal benefit accruing from those resources remains constant. In standard models, with exponential discounters, this characteristic is expressed by the well known Euler condition [2],

$$u'(c_t) = \delta r_{t+1} u'(c_{t+1})$$

This states that an agent is indifferent between spending an additional unit of resource in the current period or saving it until the subsequent period. The benefit of current spending is given by the marginal utility of consumption. The benefit of saving is given by the marginal utility of consumption in the subsequent period, augmented by capital growth, but diminished by inter-temporal discounting.

[1] The term 'welfare enhancing' is used here in the narrow sense of welfare as perceived by the decision maker at that point in time when the decision is made.

[2] In this paper I consider a perfect foresight model, so expectations are ignored.

Under hyperbolic discounting, the general form of the Euler condition is preserved, but the discount factor is altered. In the quasi-hyperbolic model, there are two distinct discount rates. The long-run rate is denoted δ, and is used to discount utility between subsequent periods at any point in the future. The short run rate, $\beta\delta$ is used to discount from the next period to the current period. Harris and Laibson (2003) show that in an endowment economy, and in the absence of commitment mechanisms, the Euler condition becomes:

$$u'(c_t) = \{\beta\delta C'(b_{t+1}) + \delta(1 - C'(b_{t+1}))\} r_{t+1} u'(c_{t+1})$$

The effective discount rate, in the curly brackets, is a weighted average of the short-run and long-run discount rates. The weight on the short-run rate is the subsequent period's marginal propensity to consume out of wealth. This arises from the time inconsistency inherent in hyperbolic discounting. The agent at time one (self 1) values consumption at time 1 relative to consumption at time 2 more highly than the agent at time zero (self 0), and will therefore overconsume at time 1 from self 0's perspective. Therefore, self 0 values marginal saving at $t \geq 1$ more highly than marginal consumption at $t \geq 1$. Hence, self 0 values the future less the higher the marginal propensity to consume out of wealth at $t = 1$.

Further complexity is added to the Euler condition when we consider the case of a quasi-hyperbolic agent in a productive economy. I derive the Euler condition for such an economy, assuming again that no commitment mechanisms are available, in appendix A. It is given by:

$$u'(c_t) = \{\beta\delta\Gamma + \delta(1 - \Gamma)\} r_{t+1} u'(c_{t+1})$$

$$\text{where } \Gamma = C'(b_{t+1}) + \frac{u'(w_{t+1})}{u'(c_{t+1})} W'(b_{t+1})$$

In this case the weight on the short-run rate is not only a positive function of the marginal propensity to consume out of wealth in the subsequent period. It is also a positive function of the marginal propensity to shirk with increasing wealth (to increase wages and thus reduce labour supply) in the subsequent period. The intuition here is again based on the time inconsistent choices of the agent. From the perspective of self 0, self 1 overvalues leisure at time 1. Therefore, self 0 values the future less the more self 1 takes advantage of extra initial wealth to shirk in period 1.

It is clear from the preceding discussion that self 0 would benefit from being able to constrain the actions of future selves. Various literature (notably Laibson 1997) has considered the incentives for hyperbolic agents to engage commitment mechanisms, such as the acquisition of illiquid assets, which limit the ability of their future selves to consume out of accumulated wealth. Such commitment mechanisms constrain the marginal propensity to consume out of wealth in future periods, $C'(b_{t+1})$, and thus reduce the weight on the short-run discount rate in the present self's Euler condition. Intuitively, agents are more prepared to save for the future if they can prevent their future selves from squandering that accumulated wealth.

In this paper, I consider an alternative commitment mechanism which will be attractive to hyperbolic discounters who wish to constrain the behavior of their future selves. This is a commitment to future wage rates. The rational hyperbolic agent recognizes that if it bequeaths a large amount of wealth to its future self, then its future self will shirk to a greater extent than self 0 deems optimal. This discourages self 0 from accumulating wealth to the extent that it otherwise would. Committing its future self to a particular wage level in future periods means that $W'(b_{t+1}) = 0$, and reduces the weight on the short-run discount rate in the present self's Euler condition.

3 The Model

I consider a dynamic general equilibrium model under perfect foresight. It consists of four agents. The representative firm employs differentiated labour from two cohorts of households, and transforms that labour into a final good which it sells in a perfectly competitive market in order to maximize its profit.

The government issues one period bonds, setting the nominal interest rate so that inflation is equal to a target π^*.

Barro (1999) shows that in a representative agent model with no nominal rigidities, hyperbolic and exponential discounting are observationally equivalent. Although hyperbolic discounters would like to bring consumption forward to the present period, in a representative agent model this is not possible because there is no one else to borrow from. The effect is simply to drive up the interest rate, and this is observationally equivalent to an economy with exponential discounters whose discount rate is higher than the long-run rate of the hyperbolic agent.

In order to allow the hyperbolic agents scope to borrow I inhabit this model with two cohorts of households. The first representative household has exponential time preferences and only the second has hyperbolic preferences.

Each household can choose to whether to supply its labour

· flexibly, re-optimizing its wage rate each period;

· according to a binding contract which fixes the real wage for all subsequent periods; or
· according to a binding contract which fixes the nominal wage for all subsequent periods.

For reasons of tractability, I limit the model to three time periods. Given that all resources are consumed in the final time period, three is the minimum number of time periods that are required to allow the time-inconsistencies inherent in hyperbolic discounting to emerge.

3.1 Production

The representative firm faces a Dixit-Stiglitz (1977) production technology. It combines the labour of the various households, $l_t(i)$, together to produce a homogenous good, y_t, according to:

$$y_t = \left[\sum_{i=e,h} \frac{1}{2} \left(l_t(i)^{\frac{\theta-1}{\theta}} \right) \right]^{\frac{\theta}{\theta-1}}$$

where θ is the elasticity of substitution between the labour of the two cohorts.

The firm's profit maximization implies that each household faces the following demand for its labour service:

$$l_t(i) = w_t(i)^{-\theta} y_t$$

where $w_t(i)$ is the real wage set by households in cohort i for period t.

3.2 Household Decision Making

Each household's intra-period utility function is given by:

$$u\left(c_t(i), l_t(i)\right) = \ln\left(c_t(i)\right) - \frac{l_t(i)^{1+\phi}}{1+\phi}$$

Households can transfer wealth from one period to the next by holding government bonds, $B_t(i)$, which earn a gross return R_t. Therefore, they face the following budget constraint:

$$c_t(i)P_t + B_{t+1}(i) = W_t(i)l_t(i) + T_t(i) + R_t B_t(i)$$

where $T_t(i)$ is a lump-sum transfer from the government of the proceeds from seigniorage.

Household e has an exponential inter-temporal discount function. The discount factor is δ_e between each time period and its subsequent period:

$$U_0(e) = \sum_{t=0}^{2} \delta_e^t u\left(c_t(e), l_t(e)\right) \tag{1}$$

Household h has a quasi-hyperbolic (see Laibson 1997) inter-temporal discount function. The discount factor is $\beta\delta_h$ between the current and the following period, but is equal to δ_h between any future time period and its subsequent period:

$$U_0(h) = u\left(c_0(h), l_0(h)\right) + \beta \sum_{t=1}^{2} \delta_h^t u\left(c_t(h), l_t(h)\right) \tag{2}$$

This gives rise to time inconsistent behavior. Household h will have incentives in the future to change plans that have been made in the current period. A rational household will recognize the problem that it cannot commit itself to implementing any plan beyond the current period and so it will adapt its current behavior to take this into account.

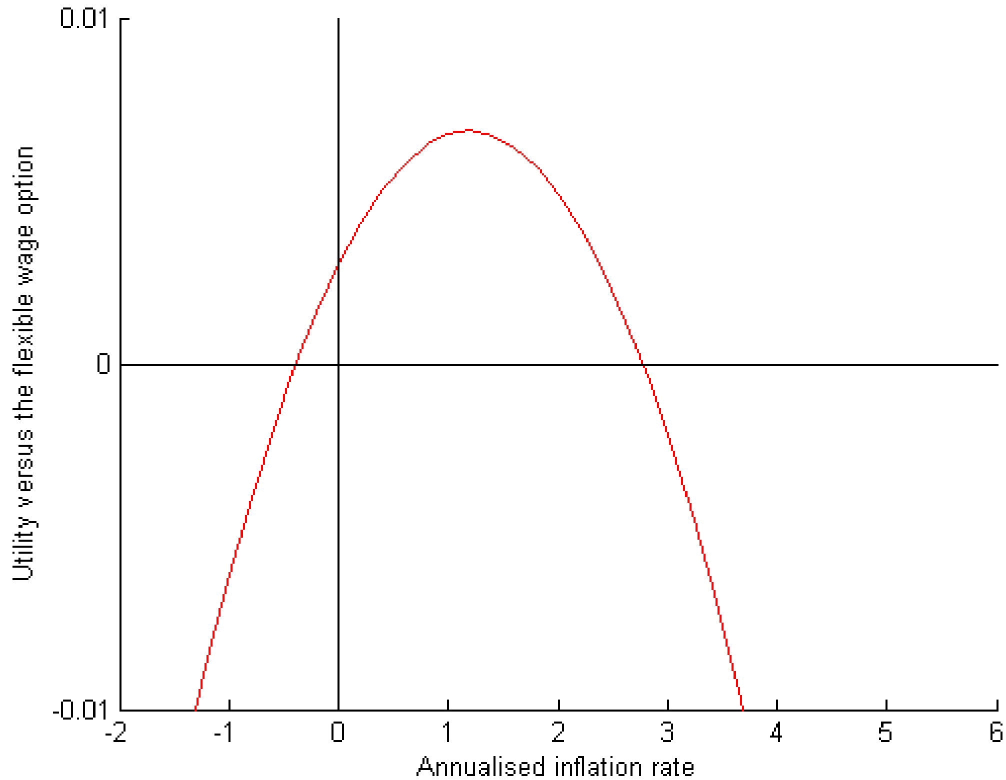
Each household must decide whether to sign a binding multi-period nominal or real wage contract, or to supply its labour flexibly. It must then choose the wage that it sets. As is conventional in the New Keynesian literature, it is assumed that the household will then supply as much labour as is demanded at that wage.

The problem that household h faces incorporates strategic interaction. Time inconsistency implies that the household is engaged in a strategic game with itself. If it chooses to engage a commitment mechanism then this is because it is trying to constrain its own future behavior, and so must rely on some conjecture about the nature of that future behavior. I therefore (following Laibson 1997) model the problem facing the household as a game played between the household in the current period and itself in future periods.

Household e does not face any time inconsistency problems and so the flexible wage contract will always be optimal for this household.

For household h, however, a fixed wage contract represents a commitment mechanism which it can employ to affect some control over the behavior of its future self, and ameliorate the problems associated with time inconsistency.

Fig. 1.



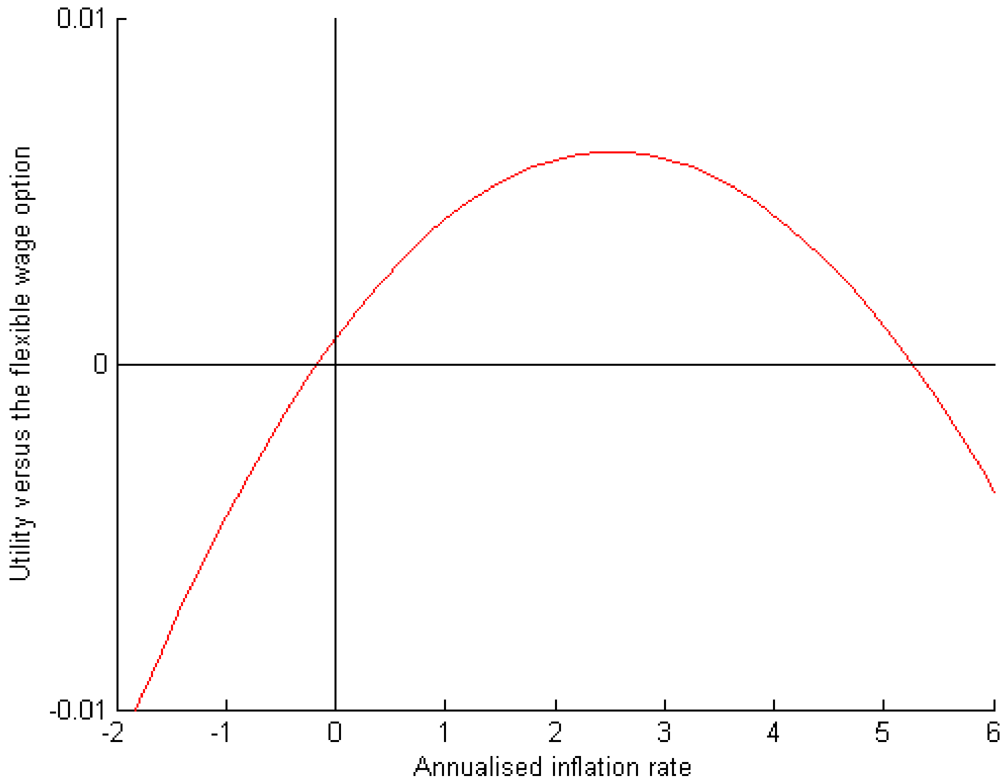
4 Results and Conclusions

The results from the base calibration of the model are illustrated in figure 1. The horizontal axis at zero represents the lifetime utility of household h as assessed by itself in period 0, given that it follows a strategy of reoptimizing each period. In other words, this is utility conditional upon signing a flexible wage contract. The red line then illustrates the lifetime utility of household h as assessed by itself in period 0, given that it ties itself into a fixed nominal wage contract in period 0 which is effective for all three periods. In this model, a fixed nominal wage contract when the general inflation rate is zero is exactly equivalent to a fixed real wage contract.

We can see from figure 1 that a fixed real wage contract is certainly superior to a flexible wage contract, as assessed by the household at time zero. For annual inflation rates up to around 2.5%, the household will find it preferable to sign a fixed nominal wage contract.

The intuition for this result is that, by tying itself into a contract which involves a modestly declining real wage, the household is able to commit its

Fig. 2.



future self to work harder. This in turn reduces the effective discount rate of the household and makes saving from period 0 more attractive.

Figure 2 illustrates one other calibration of the model, in which the relative sizes of the cohorts have been altered. Under the base parameterization, both households accounted for 50% of the total population. In the calibration that underlies figure 2, the exponential cohort accounts for 90% of the population whilst the hyperbolic discounters only account for 10%. The attractiveness of fixed nominal wages is even greater in this case.

References

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[3] Harris, Christopher and David Laibson, 2003. "Hyperbolic Discounting and Consumption," Advances in Economics and Econometrics, Cambridge University Press.

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Appendix A:

Solving the Hyperbolic Discounting Model as a Dynamic Programming Problem [3]

Preferences of the hyperbolic household are described as follows:

$$U\left(c_t, l_t\right) + \beta \sum_{s=t+1}^{\infty} \delta^{s-t} U\left(c_s, l_s\right) \tag{3}$$

Due to the time inconsistent nature of preferences, this model cannot be put into stationary recursive form in the usual manner. Instead, I define a future-value function, V, and a current-value function, T, where initial bond holdings are the state variable.

$$V\left(b_{t+1}\right) = U\left(C\left(b_{t+1}\right), W\left(b_{t+1}\right)\right) + \delta V\left(b_{t+2}\right) \tag{4}$$

$$T\left(b_t\right) = U\left(C\left(b_t\right), W\left(b_t\right)\right) + \beta\delta V\left(b_{t+1}\right) \tag{5}$$

Consumption and wages are the two control variables, and the (possibly time varying) policy functions relating to these control variables are $C\left(b_t\right)$ and $W\left(b_t\right)$ respectively. I am assuming here that the household recognizes the symmetry of their problem across time, and so only considers policy functions that are time invariant. Given that allocations are made by the self at time t with an instantaneous discount factor $\beta\delta$, the policy functions must satisfy:

$$\left\{C\left(b_t\right), W\left(b_t\right)\right\} \in \arg\max_{c,w} \left[U\left(c, w\right) + \beta\delta V\left(b_{t+1}\right)\right] \tag{6}$$

It follows from the definitions of V and T that:

$$T\left(b_{t+1}\right) = \left(1-\beta\right) U\left(C\left(b_{t+1}\right), W\left(b_{t+1}\right)\right) + \beta V\left(b_{t+1}\right) \tag{7}$$

[3] In this appendix, I adopt the convention of representing all functions by capital letters, all variables by lower case letters, and parameters by lower case Greek letters.

Wealth, held in the form of bonds, is given by:

$$b_{t+1} = B\left(b_t, c_t, w_t\right) \tag{8}$$

assuming that aggregate output is treated as exogenous by the household, so that labour income depends only on the wage rate.

In what follows, for ease of notation, I drop the time subscripts and represent subsequent period variables with a $\widetilde{\ }$.

The first order conditions associated with equation (6) are as follows:

$$\frac{\partial U\left(c,w\right)}{\partial c} + \beta\delta V'\left(\tilde{b}\right)\frac{\partial B\left(b,C\left(b\right),W\left(b\right)\right)}{\partial c} = 0 \tag{9}$$

$$\frac{\partial U\left(c,w\right)}{\partial w} + \beta\delta V'\left(\tilde{b}\right)\frac{\partial B\left(b,C\left(b\right),W\left(b\right)\right)}{\partial w} = 0 \tag{10}$$

Differentiating the current-value function (5) gives:

$$\begin{aligned} T'\left(b\right) = {} & \frac{\partial U\left(C\left(b\right),W\left(b\right)\right)}{\partial c}C'\left(b\right) + \frac{\partial U\left(C\left(b\right),W\left(b\right)\right)}{\partial w}W'\left(b\right) \\ & + \beta\delta V'\left(\tilde{b}\right)\left\{\begin{array}{c} \frac{\partial B(b,C(b),W(b))}{\partial b} \\ +\frac{\partial B(b,C(b),W(b))}{\partial c}C'\left(b\right) \\ +\frac{\partial B(b,C(b),W(b))}{\partial w}W'\left(b\right) \end{array}\right\} \end{aligned}$$

which simplifies to the following when conditions (9) and (10) are substituted in:

$$T'\left(b\right) = \beta\delta V'\left(\tilde{b}\right)\frac{\partial B\left(b,C\left(b\right),W\left(b\right)\right)}{\partial b}$$

Note from the budget constraint (8) that:

$$\frac{\partial B\left(b,C\left(b\right),W\left(b\right)\right)}{\partial b} = -\frac{\partial B\left(b,C\left(b\right),W\left(b\right)\right)}{\partial c}$$

Therefore:

$$\begin{aligned} T'\left(b\right) &= -\beta\delta V'\left(\tilde{b}\right)\frac{\partial B\left(b,C\left(b\right),W\left(b\right)\right)}{\partial c} \\ &= \frac{\partial U\left(c,w\right)}{\partial c} \end{aligned} \tag{11}$$

which is the standard envelope condition.

Differentiating equation (7) gives:

$$T'(b) = (1-\beta)\left\{\frac{\partial U(C(b), W(b))}{\partial c}C'(b) + \frac{\partial U(C(b), W(b))}{\partial w}W'(b)\right\} + \beta V'(b) \tag{12}$$

Substituting (11) into (12), forwarding by one period, and multiplying both sides by δ:

$$\beta\delta V'\left(\widetilde{b}\right) = \delta\frac{\partial U(\widetilde{c}, \widetilde{w})}{\partial \widetilde{c}} - (1-\beta)\,\delta\left\{\frac{\partial U(\widetilde{c}, \widetilde{w})}{\partial \widetilde{c}}C'\left(\widetilde{b}\right) + \frac{\partial U(\widetilde{c}, \widetilde{w})}{\partial \widetilde{w}}W'\left(\widetilde{b}\right)\right\} \tag{13}$$

Finally, substituting (13) into (9) gives the following Euler condition:

$$\frac{\partial U(c, w)}{\partial c} = -\frac{\partial B(b, C(b), W(b))}{\partial c}\left[\delta(1-\Gamma) + \beta\delta\Gamma\right]\frac{\partial U(\widetilde{c}, \widetilde{w})}{\partial \widetilde{c}}$$

$$\text{where } \Gamma = C'\left(\widetilde{b}\right) + \frac{\frac{\partial U\left(\widetilde{c}, \widetilde{w}\right)}{\partial \widetilde{w}}}{\frac{\partial U\left(\widetilde{c}, \widetilde{w}\right)}{\partial \widetilde{c}}}W'\left(\widetilde{b}\right)$$